Updating of chloride ingress models using core data measurements

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Abstract: A large proportion of concrete bridges in North America are reaching the end of their service life. An important factor that accelerates deterioration is the amount of salts used during winter months. The chloride ions from the salt enter the concrete and destroy the passive-oxide layer that protects the reinforcing steel from corrosion. The TransChlor® software is used to simulate the time history of chloride ion ingress into a typical concrete bridge. A climate model is used to simulate the exposure of the structure to winter conditions. A probabilistic method is used to propagate the uncertainty on material properties in the model and to obtain the distribution of chloride ion concentrations. A procedure for updating the distribution is proposed using data from core samples at two depths (25/50 mm) and for the two types of exposure (splash/direct). The procedure is shown to be very efficient in reducing bias and variance from model predictions.

Keywords: concrete bridges; deterioration; initiation of corrosion; updating; propagation of uncertainty.
1 Introduction

Degradation of concrete bridge structures is a major concern in North America and a large proportion of repair costs are directly related to corrosion of reinforcing steel. As a result, many bridges require frequent inspections, repairs, retrofitting or decommissioning. With limited resources, it is crucial that maintenance strategies and rehabilitations are planned optimally. To achieve this, designers, engineers and owners must be better informed on the rate of deterioration of bridges and its effect on reliability (USDOT, 2006; USDOT, 2010; ASCE, 2013; Davis and Goldberg, 2013).

Models are required to develop optimal maintenance/replacement strategies that account for all phases of deterioration of structures during their service life. There are many different modes of deterioration of concrete structures: sulphate attack on the cement paste, alkali–silica reaction of the aggregates, freeze-thaw cycles, corrosion of the reinforcing steel, etc. Among these, corrosion of reinforcement is the most common and important degradation mechanism for concrete bridges and viaducts.

During the winter months, de-icing salts are spread across the road surface to promote safe ‘bare pavement’ driving conditions (U.S. Roads, 1997; Transportation Association of Canada, 2003; Environment Canada, 2006; Salt Institute, 2009; Environment Canada, 2010). These salts contain chloride ions which destroy the passive oxide layer that protects the reinforcing steel from corrosion. Accurately predicting the deterioration processes due to the presence of chloride ions is a crucial component in the development of optimal maintenance strategies. In this application, a model that simulates the propagation of chloride ions in concrete is used to predict the time of initiation of corrosion.

Figure 1 Rates for sodium chloride road salts, 1998: (a) Minimal, maximal and average provincial application and (b) Average municipal application

Source: Morin and Perchanok (2000)
The policy of ‘bare pavement’ has been widely adopted by most cities in North America. As a result, salt usage doubled every five years during the 1950s and 1960s growing from 1 million tons in 1955 to nearly 10 million tons less than 15 years later (National Research Council, 1991). Figure 1 (a) shows typical average use of de-icing salts on roadways in Canadian provinces (between 100 and 350 kg/2-lane km for Quebec). Figure 1 (b) shows the average use of de-icing salts on municipal roads (300 kg/2-lane km for Montreal), which tends to be higher than provincial averages. Critical bridges and highways are typically treated with higher application rates and more frequent treatments. Lower-priority streets and secondary roads are often left untreated for longer periods of time or not treated at all (National Research Council, 1991). The use of high volumes of de-icing salts is detrimental to the durability of the structures but their use is required to maintain a high level of safety on roadways.

2 Modelling exposure and chloride ions transport in concrete

When reinforcing steel is placed in concrete, the high pH of concrete creates an oxide film on the surface of the steel. The corrosion process of the reinforcing steel begins once this passive oxide layer is destroyed. Reactions with chloride ions, as well as the presence of moisture and oxygen, contribute to the destruction of the oxide film. The concentration of chloride ions required to destroy the oxide film is referred to as the critical or threshold chloride ion concentration. Carbonation of the concrete also destroys the oxide film by lowering the pH of the concrete (Huang and Yang, 1997; Enright and Frangopol, 2000; Stewart and Rosowsky, 1998; Akgul and Frangopol, 2005a; Akgul and Frangopol, 2005b; Val and Stewart, 2005; Duprat, 2007). The critical chloride concentration is highly variable owing to the numerous factors that influence this process. These factors include the concrete mix type and proportions, pH, C3A content, water to cement ratio, relative humidity, and temperature (Stewart and Rosowsky, 1998; Duprat, 2007). Owing to the variability in concrete properties, the critical chloride concentration can vary between 0.2% and 3% by weight of cement (Duprat, 2007).

The chloride diffusion coefficient (D\text{Cl}) represents the amount of chloride ions diffusing into the concrete as a function of time. This coefficient varies widely and is influenced by parameters such as mix proportions, curing time and conditions, compaction, environmental exposure and time effects (Stewart, 2003). A low chloride diffusion coefficient significantly increases the time until the initiation of corrosion (Kirkpatrick et al., 2002). The surface chloride concentration is influenced by the amount of de-icing salts applied at the surface, as well as by the drainage efficiency and the water tightness of expansion and construction joints (Stewart, 2003). The chloride ion concentration in the concrete decreases in the spring, summer, and fall when no salts are applied onto the surface and increases in the winter when de-icing salts are used (Kirkpatrick et al., 2002).

Figure 2 Profile of a bridge with different exposure zones

The most basic transport models are based on the simple diffusion model. Improvements of this model include capillary suction, aqueous water transport with and without chlorides, thermal transfer, vapour transfer, carbon dioxide diffusion, etc. Some models, such as Stadium\textsuperscript{®} (Samson and Marchand, 2007; Marchand and Samson, 2009), include advanced chemical interactions and change within the concrete, while others focus more on the environmental and micro-climate around the structures, such as TransChlor\textsuperscript{®} (Conciatori, 2005; Conciatori et al., 2010). A few models also account for cracking of the concrete (Jefremczuk, 2005). To date, however, there is not a single model that accounts simultaneously for complex chemical reactions, environmental conditions, and cracking of concrete. The model used in this work is the TransChlor\textsuperscript{®} model with its micro-climate model. TransChlor\textsuperscript{®} was preferred since the micro-climate and history of local weather events greatly influence the process of ingress of chloride ions into the concrete elements (i.e.
rain, snow, humidity, sunny, etc.) and can account for various exposure conditions (direct, splash or mist).

The micro-climate model considers the climatic environment of the various elements of a structure. It is a function of the exposure of the structural element to solar radiation, carbon dioxide concentration and weather conditions (Conciatori et al., 2010). The model distinguishes between shaded areas, sunny areas, areas exposed to precipitation water, protected areas, areas of rapid drying and areas of stagnant water. The micro-climate parameters are comprised of the air temperature, relative humidity of air, precipitation, and stagnation of water. The severity of the winter period is defined as the number of days where the hourly temperature never goes above 0°C. Wind intensity and direction are not directly considered in the model. The effect of wind is indirectly accounted for through calibration with field data (Conciatori et al., 2009a). On a bridge, the exposure of the structural elements is classified into three categories: (i) direct, (ii) splash, and (iii) mist exposures (Figure 2).

For this application, hourly climate data were obtained for a period starting at the beginning of the service life (1965) of a typical older bridge in Montreal until present (Figure 3).

The TransChlor® model accounts for two primary chemical reactions: carbonation and adsorption of chloride ions by the cement paste. The second chemical reaction is a reversible transformation and therefore can absorb or release chloride ions into the concrete pore structure. The TransChlor® model assumes a one-dimensional linear transport of chlorides into the concrete. The model uses a finite element method to solve for the ion propagation within the concrete cover and a finite difference method to solve for the progression as a function of time. The transport modes include thermal and vapour transfers, liquid water transport with and without chloride ions, capillary suction, chloride ion diffusion in water, and carbon dioxide diffusion in concrete. Movements of substances in the concrete depend directly on the concrete permeability for all transport modes, except for thermal transfer. The time step used in the simulation is one hour. This time interval provides the required level of accuracy in the simulation for determining temperature profiles, precipitation history and wetting/drying cycles (Conciatori et al., 2008; Conciatori et al., 2010).

**Figure 3** Extract of climate data for Montreal (Environment Canada)

![Graphs of climate data for Montreal](image)

The general formula, used in the simulation, takes into account all transport modes: thermal diffusion, carbonation, hydrous transport and chloride ion transport (equation (1)).

The first term in the formula performs calculations according to Fick’s law of diffusion, while the second term is tailored for capillary suction. The parameter \( z \) in equation (2) is for the carbonation reaction (Conciatori et al., 2010).
\[
\frac{\partial \frac{[CO_2]}{H}}{\partial t} + \text{div} \left( A \left( \begin{array}{c}
\frac{\text{grad}(T)}{\text{grad}[CO_2]} \\
\frac{\text{grad}(T)}{\text{grad}(H)} \\
\frac{\text{grad}(C(B,T))}{\text{grad}(H)}
\end{array} \right) \right) = E \left( \begin{array}{c}
\frac{\text{grad}(T)}{\text{grad}[CO_2]} \\
\frac{\text{grad}(T)}{\text{grad}(H)} \\
\frac{\text{grad}(C(B,T))}{\text{grad}(H)}
\end{array} \right) - z
\]

\[z = - \left( e_{\text{ini}} \cdot f_w \cdot r_{CH} + 3 \cdot r_{CSH} + 3 \cdot r_{C3S} + 2 \cdot r_{C2S} \right)\]

\[t \quad \text{time (s)}\]

\[T \quad \text{temperature (°C)}\]

\[[CO_2] \quad \text{carbon dioxide molar concentration (mol/m}^3 \text{ air)}\]

\[H \quad \text{relative humidity in the concrete pores (—)}\]

\[C \quad \text{total Cl}^- \text{ concentration with respect to the concrete volume (kg/m}^3 \)]

\[B \quad \text{carbonation extent constant}\]

\[A \quad \text{matrix composed of the various diffusion and other coefficients}\]

\[E \quad \text{matrix composed of the convection and other coefficients}\]

\[z \quad \text{a carbonation parameter}\]

\[e_{\text{ini}} \quad \text{initial porosity prior to hydration and carbonation (—)}\]

\[f_w \quad \text{ratio of pore and film water volume to the pore volume (—)}\]

\[r_{CH} \quad \text{carbon dioxide reaction rate with portlandite (mol/s m}^3 \text{ concrete)}\]

\[r_{CSH} \quad \text{mean tricalcium and dicalcium silicate forming calcium silicate hydrates (mol/s m}^3 \text{ concrete)}\]

\[r_{C3S} \quad \text{tricalcium silicate reaction rate (mol/s m}^3 \text{ concrete)}\]

\[r_{C2S} \quad \text{dicalcium silicate reaction rate (mol/s m}^3 \text{ concrete)}\]

The matrix \(A\), (equation (3)), is composed of the various Fick’s diffusion law coefficients. The matrix contains coefficients for thermal, carbon dioxide, vapour and chloride ion diffusion, as well as the relationships between vapour and chloride ion diffusion.

\[
A = \begin{bmatrix}
\frac{\lambda_T(T,f)}{c_T(w)} & 0 & 0 & 0 \\
0 & \frac{D_{HR}(e,H_{\text{ext}})}{e(1-f)} & 0 & 0 \\
0 & 0 & D_{B,C}(T,H) & 0 \\
0 & 0 & R_{C3S}c_f(B) & D_{Cl}\end{bmatrix}
\]

where

\[\lambda_T \quad \text{concrete thermal conductivity (W/(m K))}\]

\[f \quad \text{water content with respect to the water density (—)}\]

\[c_T \quad \text{unit concrete heat-storage capacity (kJ/(m}^3 \text{ K))}\]

\[w \quad \text{concrete water content per cubic meter of concrete (kg/m}^3 \)]

\[D_{B,C} \quad \text{carbon dioxide diffusion coefficient in concrete (mm}^2\text{/s)}\]

\[e \quad \text{pore volume with respect to the total concrete volume (—)}\]

\[H_{\text{ext}} \quad \text{average atmospheric relative humidity between the concrete fabrication and current simulation time (—)}\]

\[D_{HR} \quad \text{water vapour diffusion coefficient (mm}^2\text{/s)}\]

\[R_{C3S} \quad \text{delay coefficient}\]

\[c_f \quad \text{free chloride ion concentration in the concrete interstices with respect to the solution volume (kg/m}^3 \)]

\[D_{Cl} \quad \text{free chloride ion diffusion coefficient (mm}^2\text{/s)}\].
The matrix $E$, (equation (4)), is composed of the expressions for the material movement equations, i.e. the liquid water movement and the suction of chloride ions by liquid water.

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & D_{\text{cap}} & 0 & 0 \\ 0 & R_{Ct} \cdot c_j (B) \cdot D_{\text{cap}} & 0 & 0 \end{bmatrix}$$

(4)

where $D_{\text{cap}}$ is the capillarity coefficient (mm/s). For more information on each variable and algorithms refer to Conciatori et al. (2010).

The exposure model used in the TransChlor® simulations is illustrated in Figure 4. When a rain event occurs, the concrete surface is saturated with water and the relative humidity reaches 100%. The model assumes that direct exposure starts instantaneously and that splash exposure is delayed $t_1$ hours to account for water accumulation. Following a precipitation event, the model assumes that the surface is dry $t_2$ hours after the end of precipitations. Splash exposure is assumed to end with the end of precipitations due to drainage. Mist exposure is not directly related to precipitation events but to the relative humidity. De-icing salts are applied to the road surface if any of the following four conditions are met:

1. $(T \leq T_{\text{lim}}) + (H \geq H_{\text{lim}})$
2. Rain + $(T \leq T_{\text{lim}})$
3. Snow
4. $t_{\text{event}} \geq t_{\text{interval}}$

where $T$ is the hourly temperature (°C), $T_{\text{lim}}$ is the limiting temperature (°C), $H$ is the relative humidity, $H_{\text{lim}}$ is the limiting relative humidity in the simulation, $t_{\text{event}}$ is the duration of an event (hrs), and $t_{\text{interval}}$ is the time interval between two passes of a salt truck (hours). The first condition implies that an intervention is triggered if the hourly temperature is less than a limiting temperature and that the relative humidity is greater than a limiting relative humidity. The limiting relative humidity is defined to be 95% since the surface is close to being saturated and can result in freezing of the roadway. This limiting temperature must be less than or equal to 5°C, since that is the maximum temperature when snow and freezing precipitation can occur. The second condition for an intervention occurs when there is precipitation and the temperature is less than the limiting temperature. The third condition takes place when there is snow. The last condition implies that if the duration of a winter event that required an intervention is longer than the time interval specified between the passages of de-icing trucks, another intervention is triggered.

Figure 4 Exposure types and periods for TransChlor®

The program also incorporates a probabilistic analysis of up to four concrete properties in the simulation. The four properties that can be considered by the TransChlor® model are water vapour transport ($D_{\text{HR}}$), liquid water transport by capillary suction ($D_{\text{CAP}}$), chloride ion transport ($D_{\text{CL}}$), and carbonation. The model uses a two-point Rosenblueth method to propagate the uncertainty and estimate probabilistic moments for chloride ion concentrations as a function of time and position (Conciatori et al., 2009a). Numerical integration methods or Monte Carlo modelling are possible; however, they are computationally too demanding for this model (Conciatori et al., 2009b; Conciatori et al., 2014a). The Rosenblueth point estimators method discretises the distribution of input variables at specific points. These discrete points are a function of the statistical moments of the input variables. Details of the procedures and comparisons with results from Monte Carlo simulations are presented in Conciatori et al. (2014a). Considering a real function $Y$ of a real variable $X$, it can be shown (Rosenblueth, 1981) that the estimation of $n$-th moment of $Y$ can be calculated from point estimators of $X$. 
independently of the distribution of the variable. The point estimators of $X$ ($X_+$ and $X_-$) and their associated weights ($p_+$ and $p_-$) are defined as:

$$X_+ = m_x + \zeta_+ \sigma_x \quad \quad X_- = m_x + \zeta_- \sigma_x$$

$$p_+ = \frac{\zeta_+}{\zeta_+ + \zeta_-} \quad \quad \quad \quad p_- = 1 - p_+$$

where

$$\zeta_+ = \frac{\gamma_x}{2} \sqrt{1 + \left( \frac{\gamma_x}{2} \right)^2} \quad \quad \zeta_- = \zeta_+ - y_x$$

and $m_x$, $\sigma_x$ and $\gamma_x$ are the mean, standard deviation and skewness of $X$, respectively. In the case where $Y$ is a function of $r$ independent variables, a set of two-point estimates are defined for each variable. An estimate for the $n$-th moment of $X$ is obtained by considering simulation result ($y_i$) from all possible combinations ($c = 2^r$) of the two-point estimates for the transport properties,

$$Y^n = \sum_{i=1}^{\tilde{c}} P_i \cdot y_i$$

$$y_i = f\left( X_{\delta_1}, X_{\delta_2}, \ldots, X_{\delta_r} \right)$$

$$P_i = \prod_{j=1}^{r} P_{\delta_j}$$

where $\delta_j$ is the identifier (+, -) of the point estimate for variable $j$, and $P_i$ is the probability associated with a given combination (out of $2^r$ combinations) of transport properties.
3 Application to bridges in Montreal

A large proportion of the bridges in Montreal were constructed between 1960 and 1970. Materials used in that time period were predominantly cement type I at 350 kg/m$^3$ with water/cement ratios of 0.5, an air content of 1.5% of mass of cement, and limestone aggregates. Core samples were analysed for chloride ion content at 25-mm and 50-mm depths on several structures on elements exposed to two exposure conditions (direct and splash) after 40 years of service.

The input parameters that control salt spreading in the simulations are the average quantity of sodium chloride spread per intervention (g/m$^2$), and the time between two spreading operations ($t_{\text{interval}}$). The application rate of sodium chloride per intervention was assumed to be 140 kg/Lane-km (Morin and Perchanok, 2000; City of Toronto, 2004) which is the mean application rate for expressways and bare pavement conditions. The mean value matches the recommended application rate provided in the CEPA Priority Substance List Assessment Report (Environment Canada, 2000). Figure 5 illustrates the influence of the application rate of sodium chloride on chloride ion concentration as a function of depth from the exposed surface after 45 years of exposure for a structure in Montreal (Wolofsky, 2011). Results from measurements on core samples are compared with predictions for direct and splash exposures at depths of 25 mm and 50 mm (Table 1). The results indicate that measurements of chloride ion content are quite variable. This variability can be attributed mainly to the variability of concrete properties and local conditions, such as drainage, that are not included on the model.

Figure 5 Comparison of core sample data with chloride ion distribution curves after 45 years of service

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Chloride ion content from core samples</th>
</tr>
</thead>
</table>

a) Direct exposure  
b) Splash exposure
The time between interventions was assumed to be eight hours. The simulation was programmed to simulate exposure for 45 years of service life using climatic data provided by Environment Canada at hourly increments for Montreal. The thickness of the structural member from the exposed surface is specified at 200 mm with 200 elements for the simulations.

A probabilistic analysis is performed to propagate the uncertainty on material properties through the model. The TransChlor® model allows the user to consider up to four variables for concrete properties in the probabilistic approach: (1) the hydraulic transport of water by diffusion ($D_{HR}$), (2) the hydraulic transport of water by capillary action ($D_{CAP}$), (3) the ionic transport of chlorides ($D_{Cl}$), and (4) the carbonation ($D_{CO2}$).

The temperature of concrete influences the hydraulic transport properties ($D_{HR}$ and $D_{CAP}$), as well as the ionic transport of chlorides ($D_{Cl}$) and water concentration influences the ionic transport of chlorides ($D_{Cl}$), and carbonation, ($D_{CO2}$). These dependencies are incorporated directly in the TransChlor® software (Conciatori et al., 2010). For the probabilistic analysis, all the parameters except for carbonation were selected, since analyses indicate that the depth of carbonation for all exposure conditions did not exceed 10 mm in depth after 45 years. Therefore, since only three variables were considered, a total of eight ($2^3$) simulations were executed in this application. The exposure conditions selected for the simulations correspond to splash and direct exposures. For the probabilistic approach, mean and standard deviations for the three parameters are specified and used to compute the mean and standard deviation of the chloride ion concentration as a function of time and depth (Table 2). As discussed in Conciatori et al. (2009b) and Conciatori et al. (2014a), the lognormal distribution can be used for the latter.

### Table 2

<table>
<thead>
<tr>
<th>Core sample</th>
<th>Type of exposure</th>
<th>Chloride ion content (%) of cement</th>
<th>Chloride ion content (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>25 mm</td>
<td>50 mm</td>
</tr>
<tr>
<td>1</td>
<td>Direct</td>
<td>1.39</td>
<td>1.11</td>
</tr>
<tr>
<td>2</td>
<td>Direct</td>
<td>2.93</td>
<td>2.93</td>
</tr>
<tr>
<td>3</td>
<td>Direct</td>
<td>2.15</td>
<td>1.69</td>
</tr>
<tr>
<td>4</td>
<td>Splash</td>
<td>1.93</td>
<td>1.89</td>
</tr>
<tr>
<td>5</td>
<td>Splash</td>
<td>0.46</td>
<td>1.38</td>
</tr>
<tr>
<td>6</td>
<td>Splash</td>
<td>0.89</td>
<td>–</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>1.09</td>
<td>1.64</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td>0.77</td>
<td>0.93</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td></td>
<td>0.36</td>
<td>0.49</td>
</tr>
</tbody>
</table>

During the winter months, the concentrations of chloride ions fluctuate greatly near the concrete surface (i.e. within the breathing zone). This is due to capillary action and from alternating periods of high surface concentrations of chloride ions (during and shortly after salting operations) followed by low surface concentrations of chloride ions. Comparing the simulation results at both depths indicates that the chloride ion concentration at 25-mm depth is much more variable in time than the corresponding measurements at 50-mm depth and that predictions are dependent on meteorological conditions prior to sampling (Figure 6). The peaks in the figures correspond to salting operations, while the downward trends correspond to summer months when chloride ions slowly diffuse deeper into the concrete.
Simulation results for the eight combinations of direct and splash exposure conditions are compared to the concentrations of chloride ions from the core samples in Figure 7 and show that the observed values are within the range of simulated values at a depth of 25 mm and in the lower range of predicted values at a depth of 50 mm. The predictions are slightly above the measurements at a depth of 25 mm for splash exposure and could be attributed to a shorter period of pounding of water than assumed by the model.

The Rosenblueth point estimation method is used to compute the mean and the variance of the chloride ion concentrations at depths of 25 mm and 50 mm. In both cases, it is assumed that the distributions are lognormal (see Conciantori et al. (2014b) for a detailed discussion on the validation of the distribution). The Rosenblueth method is also used to compute the correlation between the chloride ion concentrations at the two depths. Denoting the chloride concentration at depths of 25 mm and 50 mm, respectively, $C_{25}$ and $C_{50}$, estimates of the mean, variance and correlation are obtained as,
The estimates for the parameters of the lognormal distributions at depth of 25 mm and 50 mm for direct and splash exposures are shown in Table 2. The results indicate a high level of uncertainty on chloride ion content as a function of the uncertainty on input parameters. The predicted average levels of chloride ions as well as the variance are higher at a depth of 50 mm than at a depth of 25 mm (Table 3, part 1). The correlation coefficients for predicted values at 25 mm and 50 mm are very high, indicating that the processes for chloride ion ingress are controlled by similar material properties at the two depths. The equivalent parameters for the logarithm of the chloride ion content (normally distributed) are obtained from equations (12) and (13) and are shown in Table 3 (part 2). The latter are used in the updating procedure described in the next section.

\[
\begin{align*}
\sigma^2_{\mu_{C_{25}}} &= \ln \left[ \frac{\sigma_{C_{25}}}{m_{C_{25}}} \right]^2 + 1 \\
m_{\mu_{C_{25}}} &= \ln m_{C_{25}} - \frac{1}{2} \sigma^2_{\mu_{C_{25}}}
\end{align*}
\]

(12) (13)

Table 3 Parameters for the distribution of chloride ions content

<table>
<thead>
<tr>
<th>Type of exposure</th>
<th>Direct</th>
<th>Splash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25 mm</td>
<td>50 m006D</td>
</tr>
<tr>
<td>Mean</td>
<td>10.3</td>
<td>12.1</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.6</td>
<td>7.7</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>Mean</td>
<td>2.272</td>
<td>2.329</td>
</tr>
<tr>
<td>Variance</td>
<td>0.113</td>
<td>0.337</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.99</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Notes: aLognormal distribution, bnormal distribution.

The latter can be interpreted as model-based a priori observations that can be updated on the basis of data obtained from core samples. The process of updating these distributions is presented in the next section. In this case, the updating procedure is described for various scenarios of sampling: (1) sample from a single core at one depth, (2) sample from a single core at two depths, and (3) sample from multiple cores.

4 Updating the distribution of chloride ions with core sample data

Denoting \(C_{25}\) as the concentration of chloride ions at a depth of 25 mm, the measured concentration of chloride ions \(Z_{25}\) can be related to \(C_{25}\) through either a model for additive or multiplicative errors (\(\varepsilon\)),

\[
Z_{25} = C_{25} + \varepsilon \quad \text{(14)}
\]

\[
Z_{25} = C_{25} \cdot \varepsilon \quad \text{(15)}
\]
The multiplicative error model is used in this application since both $C_{25}$ and $\varepsilon$ are assumed to be lognormally distributed. In both cases, the random variable $\varepsilon$ is used to model measurement errors and uncertainty associated with factors that are not included in the simulation model. In the multiplicative model, the mean of this variable is equal to 1 (the measurement procedure and model are unbiased) and errors in observations are proportional to the level of chloride ions present in the sample. Assuming that the variables are lognormally distributed, taking the logarithm of equation (7) results in the summation of normally distributed random variables,

$$\ln(Z_{25}) = \ln(C_{25}) + \ln(\varepsilon)$$

(16)

Maximum likelihood estimates of the distribution parameters of $\ln(C_{25})$ and $\ln(\varepsilon)$ are the sample mean and sample variance of the logarithmically transformed predictions and errors. Updates of the maximum likelihood estimates of the parameters of the distribution for the chloride ion concentration are obtained by deriving the conditional mean and conditional variance of $\ln(C_{25})$ given an observation of $\ln(Z_{25})$ from a core sample (Montgomery and Runger, 2007).

$$E[\ln(C_{25})\ln(Z_{25})] = m_{\ln(C_{25})} + \rho \frac{\sigma_{\ln(C_{25})}}{\sigma_{\ln(Z_{25})}} (\ln(Z_{25}) - m_{\ln(Z_{25})})$$

(17)

$$VAR[\ln(C_{25})\ln(Z_{25})] = \sigma_{\ln(C_{25})}^2 (1 - \rho^2)$$

(18)

where $\rho = \frac{\sigma_{\ln(C_{25})}\sigma_{\ln(C_{25})}}{\sigma_{\ln(Z_{25})}\sigma_{\ln(Z_{25})}}$ is the correlation coefficient between $\ln(C_{25})$ and $\ln(Z_{25})$. The correlation coefficient is obtained from the covariance matrix which is derived by considering the following set of linear combinations,

$$\begin{bmatrix} \ln(C_{25}) \\ \ln(Z_{25}) \end{bmatrix} = [H] \begin{bmatrix} \ln(C_{25}) \\ \ln(\varepsilon) \end{bmatrix}$$

where

$$[H] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Defining $\Sigma_{\ln(C_{25})\ln(\varepsilon)} = \begin{bmatrix} \sigma_{\ln(C_{25})}^2 & 0 \\ 0 & \sigma_{\ln(\varepsilon)}^2 \end{bmatrix}$

$$\Sigma_{\ln(C_{25})\ln(Z_{25})} = [H] \Sigma_{\ln(C_{25})\ln(\varepsilon)} [H]^T = \begin{bmatrix} \sigma_{\ln(C_{25})}^2 & \sigma_{\ln(C_{25})}\sigma_{\ln(Z_{25})} \\ \sigma_{\ln(C_{25})}\sigma_{\ln(Z_{25})} & \sigma_{\ln(Z_{25})}^2 + \sigma_{\ln(\varepsilon)}^2 \end{bmatrix}$$

$$m_{\ln(C_{25})\ln(Z_{25})} = [H] m_{\ln(C_{25})\ln(\varepsilon)} = \begin{bmatrix} m_{\ln(C_{25})} \\ m_{\ln(Z_{25})} \end{bmatrix}$$

(19)

where $\Sigma$ denotes the covariance matrix and $m$ denotes the mean value vector. The chloride ion content at 25 mm estimated with the model is initially uncertain with mean $m_{\ln(C_{25})}$ and variance $\sigma_{\ln(C_{25})}^2$. To reduce this uncertainty, a measurement $Z_{25}$ (or $\ln(Z_{25})$) is made. If measurements have no error, then $C_{25}$ (or $\ln(C_{25})$) is recovered exactly from $Z_{25}$ (or $\ln(Z_{25})$). However, in practice, measurements are noisy, either from experimental procedure or local conditions that cannot be included in the prediction model, but the measurements can be used to reduce the uncertainty on $C_{25}$ (or $\ln(C_{25})$).

Figure 8 illustrates the updating procedure for predictions at a depth of 25 mm from the exposed surface for splash exposure. Median and 95% confidence intervals obtained from the model simulations are shown as the dashed lines. The measurement from a single core sample is shown as the grey horizontal line. Updated median and confidence intervals are shown as a function of the coefficient of variation on core sample measurements. The uncertainty on core sample...
measurements are related to uncertainties associated with the measurement process as well as to local exposure parameters that are not included in the model used for simulations. Updated values correspond closely with sample measurements for a low coefficient of variation, indicating high confidence on sample values. As the coefficient of variation increases above 1, the data from the sample are less reliable and have lesser influence on the updated distribution. For coefficients of variation below 1, the updating procedure reduces the bias of the model and decreases the uncertainty of predictions.
Next, we consider the case when data from several samples are available for the same structure or interchange. In this case, several options can be considered for updating the model estimates. One option is to consider that the information at each core location as representative of local concrete and exposure conditions (splash or direct). In that case, the updating procedure is applied independently to each location and each depth. This procedure is most accurate if core samples are obtained from each structure of the same interchange. However, in the case of a large interchange, it may not be feasible to obtain samples from each structure. In that case, an alternative is to group structural elements that were sampled according to exposure and to assume that material properties and exposure conditions are similar throughout the interchange. These assumptions are tenable for conditions of direct and splash exposure. Updating with several observations at the same depth on several cores under similar exposure conditions is obtained with the vector form of the previous equation,

\[
\ln(Z_{25}) = \ln(C_{25}) + \ln(\varepsilon)
\]

where \(\varepsilon\) is a column vector of \(\varepsilon\)s,

\[
\begin{bmatrix}
\ln(Z_{251}) \\
\ln(Z_{252}) \\
\ldots \\
\ln(Z_{25n})
\end{bmatrix}
= \begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}
\begin{bmatrix}
\ln(C_{251}) \\
\ln(C_{252}) \\
\ldots \\
\ln(C_{25n})
\end{bmatrix}
+ \begin{bmatrix}
\ln(\varepsilon_1) \\
\ln(\varepsilon_2) \\
\ldots \\
\ln(\varepsilon_n)
\end{bmatrix}
\]

\[
\sigma_{\ln(Z_{25})}^2 = \sigma_{\ln(C_{25})}^2 + \sigma_{\ln(\varepsilon)}^2 \left( \left( \left( \sigma_{\ln(C_{25})}^2 \right)^{1/2} + \sum \ln(\varepsilon) \right)^{-1} \right) ^2 
\]

\[
m_{\ln(Z_{25})} = m_{\ln(C_{25})} + \sigma_{\ln(\varepsilon)}^2 \left( \left( \left( \sigma_{\ln(C_{25})}^2 \right)^{1/2} + \sum \ln(\varepsilon) \right)^{-1} \right) \left( m_{\ln(Z_{25})} - \ln(Z_{25}) \right)
\]

Figures 9 and 10 show results for the updated distribution as a function of the uncertainty on the measurements and the number of observations. The results are shown for the distribution of chloride ion content at 25–50-mm depths and for splash and direct exposures. Figure 9 shows the reduction in the variance of model predictions as a function of the uncertainty on core measurements for the cases with one and multiple samples. The reduction in the variance is significant for coefficients of variation on measurements below 0.5, when one core sample is used for updating. The reduction in the variance is improved for a given coefficient of variation by increasing the number of cores from 1 to 3. Reductions in the variance with updating are more significant at a depth of 50 mm compared to 25 mm and can be attributed to the higher level of initial uncertainty at depths of 50 mm. Finally, for the case with three samples, estimates of the coefficient of variation on measurements are between 0.2 and 0.7 (Table 2).
Figure 9 Reduction of variance as a function of the coefficient of variation on measurements and the number of samples

Figure 10 shows the effect of updating on estimates of median chloride ion concentrations. The figure compares the initial estimate of the median obtained from propagating the uncertainty into the prediction model to updated values of the median for each individual sample and for all samples considered simultaneously. Also shown in Figure 10 are the values obtained from core measurements. For low uncertainty on measurements, the updated medians are very close to the core sample data and asymptotically approach model predictions when uncertainty on measurements is large, indicating that the latter are non-informative. In the case where all core data are used simultaneously, the median is close to the median of the sample for low levels of measurement uncertainty. As the coefficient of variation increases, the information obtained from the samples is more informative than the single core cases. In this case, coefficients of variation between 0.2 and 0.7 are considered to be representative of experimental errors. For this range, the results indicate that the updating procedure decreases the bias in the original distribution obtained from simulations as well as the variance of the distribution.

Figure 10 Median of chloride ion content as a function of the COV on measurements

5 Conclusions

Models for the application of de-icing salts and the ingress of chloride ions in concrete structures are important for
predicting the time to initiation of corrosion and ultimately the service life of structures. The TransChlor® model was used to estimate the chloride ion concentrations in typical structures in Montreal built in the 1960s. Predictions were obtained by considering the detailed time history of climatological conditions during the past 50 years and salt spreading strategies for direct, splash and mist exposure conditions. Uncertainty on material properties was considered and propagated through the model by using the Rosenblueth point estimate procedure. The resulting distribution was then compared to observations obtained from core samples under splash and direct exposure conditions at depths of 25 mm and 50 mm. An updating procedure is proposed that uses observations from core sample data and is shown to greatly reduce the bias and variance on predicted ion chloride content. The procedure can be used for updating based on single or multiple observations simultaneously. Results indicate a significant reduction in bias and uncertainty, when the coefficients of variation on single observations are below 1.0 which greatly improves the accuracy for the prediction of the residual life of structures.

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References


Jefremczuk, S. (2005) Chloride Ingress and Transport in Cracked Concrete, McGill University, Montreal, p.188.


List of symbols

- \( t \) time (s)
- \( T \) temperature (°C)
- \([\text{CO}_2]\) carbon dioxide molar concentration (mol/m\(^3\) air)
- \( H \) relative humidity in the concrete pores (–)
- \( C \) total Cl\(^-\) concentration with respect to the concrete volume (kg/m\(^3\))
- \( B \) carbonation extent constant
- \( A \) matrix composed of the various diffusion and other coefficients
- \( E \) matrix composed of the convection and other coefficients
- \( z \) a carbonation parameter
- \( \varepsilon_{\text{ini}} \) initial porosity prior to hydration and carbonation (–)
- \( f_w \) ratio of pore and film water volume to the pore volume (–)
- \( r_{\text{CH}} \) carbon dioxide reaction rate with portlandite (mol/s m\(^3\) concrete)
- \( r_{\text{CSH}} \) mean tricalcium and dicalcium silicate forming calcium silicate hydrates (mol/s m\(^3\) concrete)
- \( r \), \( r \), \( r \), \( r \) tricalcium silicate reaction rate (mol/s m\(^3\) concrete)
- \( r \) dicalcium silicate reaction rate (mol/s m\(^3\) concrete)
- \( \lambda_T \) concrete thermal conductivity (W/(m K))
- \( f_w \) water content with respect to the water density (–)
- \( c_T \) unit concrete heat-storage capacity (kJ/(m\(^3\) K))
- \( w \) concrete water content per cubic meter of concrete (kg/m\(^3\))
- (kg/m\(^3\)) \( D_B \) carbon dioxide diffusion coefficient in concrete (mm\(^2\)/s)
- \( \varepsilon \) pore volume with respect to the total concrete volume (–)
- \( H_{\text{ext}} \) average atmospheric relative humidity between the concrete fabrication and current simulation time (–)
- \( D_{\text{HR}} \) water vapour diffusion coefficient (mm\(^2\)/s)
- \( R_{\text{Cl}} \) delay coefficient
- \( c_f \) free chloride ion concentration in the concrete interstices with respect to the solution volume (kg/m\(^3\))
- \( D_{\text{Cl}} \) free chloride ion diffusion coefficient (mm\(^2\)/s)