Calculation of Reinforced Concrete Corrosion Initiation Probabilities Using Rosenblueth Method

ABSTRACT

The Rosenblueth method is used to analyze stochastic parameters in a numerical model to calculate the solution for independent variables. By replacing all or a part of the independent variables with random variables, the solution becomes itself random. The Rosenblueth method simplifies this passage, while being limited to dichotomous random variables. This avoids the complex and computer intensive Monte Carlo simulation method.

A specific example is developed in this article using a nonlinear transport model to simulate the chloride ion penetration into reinforced concrete and to predict the development of corrosion in the civil structure. The making into consideration of complex physical chemical process and a reconstituted climate increase considerably the simulation time, making impossible the usual use of the Monte Carlo method. The two numerical examples show that the exposure degree affects in a dominating way the apparition of structural damage.

KEYWORDS

Probabilistic approach, chloride model, corrosion, structures exposure, concrete.

1 INTRODUCTION

Ionic transport in reinforced concrete is currently modelled with complex and nonlinear numerical methods (Marchand et al., 2003; Truc et al., 2000; Masi et al., 1997; Shin and Kim, 2002; Schmidt-Döhl and Rostasy, 1999a; Schmidt-Döhl and Rostasy, 1999b; Roelfstra et al., 2004; Tang, 1996; Meijers, 2003; Saetta et al., 1993; Maekawa and Ishida, 2000), such as the finite element method, the use of finite differences and the Fourier transformation methods (Jaun, 2003). The model parameters are seldom precisely known and are subject to large variations, resulting from measurement and modelling uncertainties. These parameters include the initial conditions, the boundary conditions and the transport model coefficients (Kirkpatrick et al., 2002; Stewart and Rosowsky, 1998; Stewart et al., 2003; Lindvall, 2001; Gehlen and Schiessl, 1999; Enright and Frangopol 2000; Zimmermann, 2000; Roelfstra et al., 2004; Conciatori, 2005).

Using random parameters modelled by a probability distribution in a complex transport model excludes all analytical solutions using direct integration. In the case of more precise numerical solutions, such as Monte Carlo simulation or other methods, enormous computing times are required (Enright and Frangopol, 1998; Enright and Frangopol, 2000; Val and Stewart, 1998; Val and Stewart, 2003; Kong, et al., 2002; Kong and Frangopol, 2005). The Rosenblueth Method or point estimation method can be viewed as a particular case of the Monte Carlo simulation method using either a stratified sampling or an antithetic variables method (Rosenblueth, 1981; Rosenblueth, 1975). Monte Carlo simulations give in result the entire random distribution, when the Rosenblueth Method gives only the first moments of the random distribution. The use of the Monte Carlo simulations is sometimes impossible,
because the method needs high iterations number. The “TransChlor” model, discussed in this article, can not be solved with the Monte Carlo method (Conciatori, 2005).

The resulting simulation employs the various temporally evolving concrete ion concentrations to represent a corrosion loading on the steel reinforcement given in this paper by “TransChlor” model (Conciatori, 2005). The convolution product between the corrosion loading and the steel reinforcement capacity to resist the ionic concentration provides the corrosion initiation probability; also known as the steel reinforcement deterioration initiation probability. The deterioration initiation probability facilitates the risk evaluation of a civil structure like a bridge, tunnel or retaining wall.

The remainder of the paper is organized as follows. In Sections 2 and 3 some general principles on the use of the Rosenblueth method are given. In Section 4, a numerical model for ion transport in reinforced concrete is described and in Sections 5 and 6 two case studies are discussed. The example, developed in this article, considers concrete low temperature capillary test results obtained in the laboratory (Conciatori et al., 2005; Conciatori and Brühwiler, 2006). Permeability parameters are represented by the various diffusivity and capillary coefficients introduced into the “TransChlor” model (Conciatori, et al., 2003; Conciatori, 2005) and thus correspond to actual reconstituted climatic and environmental exposure conditions (Conciatori et al., 2002; Denarié et al., 2003; Conciatori, 2005). The chloride ion migration in the reinforced concrete is evaluated with respect to the exposure duration and the concrete cover (location of the steel reinforcement bar in the concrete). The example focuses on the Rosenblueth method and refers to steel reinforcement corrosion initiation in concrete.

2 ROSENBLUETH METHOD

The Rosenblueth method is a simple tool for solving numerical models involving random inputs. Such problems are often encountered in questions of reliability, where the exact circumstances of the stress put onto a system are unknown. The basic idea is to approximate the actual random distributions (for example the normal or log-normal) by rough discrete distributions in which the input can only take a few distinct values with positive probability. The approximation can be chosen such that the moments of the discrete distribution are the same as those of the continuous distribution one wishes to approximate (chapter 2.1). All the combinations of the various inputs are then introduced in the model and the corresponding solutions are found (chapter 2.2). Finally, the discrete results are combined and a continuous distribution can be derived for them (chapter 2.3).

2.1 Random variables

For a random variable $X$ let its mean be $E[X] = \mu$, its variance $\text{var}[X] = \sigma^2$ and its skewness $E((X-\mu)^3) = \beta \sigma^3$ (Pearson et al., 1979). Let $Y$ be a discrete random variable concentrated at two points $x_1 < x_2$ with associated probabilities $F_1$ and $F_2 = 1 - F_1$ (Fig. 1). Matching the mean, variance and skewness of $X$ results in Equations 1 to 4 (Rosenblueth, 1975):

$$F_2 = \frac{1}{2} \left[ 1 - \frac{\beta}{|\beta|} \cdot \sqrt{1 - \frac{1}{1 + (\beta / 2)^2}} \right]$$  \hspace{1cm} (1)

$$F_1 = 1 - F_2$$  \hspace{1cm} (2)
\[ x_2 = \mu_x + \sigma_x \cdot \sqrt{F_1 / F_2} \]  
\[ x_1 = \mu_x - \sigma_x \cdot \sqrt{F_2 / F_1} \]  

These formulas are valid for any distribution and show how to compute \( x_1, x_2, \) and \( F_1 \) on the basis of \( \mu, \sigma^2 \) and \( \beta \).

Knowing the position \( x_1 \) and \( x_2 \), the associated probabilities can be represented as force vectors and can be found by the simple beam analogy (Fig. 1). The resultant force \( R \) corresponds to the surface under the distribution curve and his value is one. His position value is the mean value (Fig. 1).

The points \((x_1, x_2)\) and the associated probabilities \((F_1, F_2)\) for \( X \) with normal and log-normal distribution functions are given in Table 1.

Table 1: Positions and associated probabilities for normal and log-normal distributions. The quantities \( \alpha_m, \alpha_p \) and \( Sx \) appearing in the formulas for the log-normal are
\[ \alpha_m = e^{\xi} \cdot (1 - e^{-\lambda}), \quad \alpha_p = e^{\xi} \cdot (e^\lambda - 1) \].

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Points</th>
<th>Associated probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x_1 )</td>
<td>( x_2 )</td>
</tr>
<tr>
<td>normal ( N(\mu, \sigma) )</td>
<td>( \mu + \sigma )</td>
<td>( \mu - \sigma )</td>
</tr>
<tr>
<td>log-normal ( LN(\lambda, \xi) )</td>
<td>( e^{\xi - \lambda} )</td>
<td>( e^{\xi + \lambda} )</td>
</tr>
</tbody>
</table>

2.2 Combination of several independent random parameters

In the case of two independent random parameters \( X_1 \) and \( X_2 \) and their distributions \( P(x_1) \), \( P(x_2) \) and \( P(x_1, x_2) \), where \( P(x_1, x_2) \) denotes the joint distribution, the following equation applies:

\[ P(x_1, x_2) = P(x_1) \cdot P(x_2) \]  

Approximating \( P(x_i) \) with the two points \( x_{11} < x_{12} \) and their respective probabilities \( F_{11} \) and \( F_{12} \) and, in a similar fashion, \( P(x_2) \) by \( x_{21} < x_{22} \) and their probabilities \( F_{21} \) and \( F_{22} \), the joint distribution is concentrated on the four couples \((x_{11}, x_{21}), (x_{11}, x_{22}), (x_{12}, x_{21}) \) and \((x_{12}, x_{22})\) with the corresponding probabilities \( F_{11}F_{21}, F_{11}F_{22}, F_{12}F_{21} \) and \( F_{12}F_{22} \). These probabilities are written hereafter in the simplified form of \( Q_1, Q_2, Q_3 \) and \( Q_4 \), respectively. For \( n \) independent random variables, the joint law will be thus concentrated on \( 2^n \) n-tuples, which corresponds to the number of required simulations.

2.3 Simulation results

The support points and the probabilities found in 2.1 will be used as inputs to a numerical model. We assume that the model contains \( n \) random input variables \( X_1, \ldots, X_n \). These variables do not have precise values, only their distributions are known. The Rosenblueth
method consists in letting the variables take the values of the support points and then compute the solution. Each such solution has a weight, a probability, attached to it.

If a random parameter is substituted in a numerical model, the result itself is random. When using the Rosenblueth method, \(2^n\) combinations of parameter values are substituted into the numerical model, each having its own associated probability. Running the numerical model yields \(2^n\) results. These can be used to calculate the mean, the variance and the skewness of the random result. Eq. 6 through 8 gives the corresponding formulas for \(n\) random parameters. We have

\[
\mu_y = \sum_{k=1}^{m} Q_k \cdot y_k
\]

(6)

\[
\sigma_y^2 = \sum_{k=1}^{m} Q_k \cdot (y_k - \mu_y)^2
\]

(7)

\[
\beta \cdot \sigma_y^3 = \sum_{k=1}^{m} Q_k \cdot (y_k - \mu_y)^3
\]

(8)

with \(Q_k\) being the probability for the \(k^{th}\) combination of the \(n\) input parameters and \(y_k\) being the corresponding numerical result.

A continuous probability distribution is fitted to the results using the maximum likelihood method (Ross, 1996; Ang and Tang, 1984; Ang and Tang, 1975). Table 2 illustrates this for the normal and log-normal distributions. The choice of the appropriate probability distribution remains a primary assumption of this probabilistic model. For example, choosing a normal distribution implies a symmetrical function with a zero third moment; choosing a log-normal distribution, on the other hand, eliminates all negative value results. The log-normal distribution was chosen for the example developed in this article for the chloride ion concentrations, because this assumption has found to be appropriate for two other chloride transport models using Monte Carlo method (Enright and Frangopol, 1998; Enright and Frangopol, 2000; Kong et al., 2002; Kong and Frangopol, 2005; Marchand et al., 2008).

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability distribution parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal (f(\mu, \sigma)) (\mu_y = \sum_{k=1}^{m} Q_k \cdot y_k) (\sigma_y^2 = \sum_{k=1}^{m} Q_k \cdot (y_k - \mu_y)^2)</td>
<td></td>
</tr>
<tr>
<td>log-normal (f(\lambda, \xi)) (\lambda_y = \sum_{k=1}^{m} Q_k \cdot \ln(y_k)) (\xi_y^2 = \sum_{k=1}^{m} Q_k \cdot (\ln(y_k) - \lambda_y)^2)</td>
<td></td>
</tr>
</tbody>
</table>

### 3 COMBINATION OF DEPENDENT RANDOM VARIABLES

In many cases the input parameters of numerical models are dependent on each other and the simple formula (5) does not hold. An example is the ionic transport in concrete. The flux depends on the location and is focused on the deterioration zone, the interface between the
steel reinforcement bar and the concrete cover. The variability of the concrete cover can be modelled by a probability distribution.

The principle of total probability allows us to combine the chloride ion concentration $y$ with the concrete cover $e$ (location of the steel reinforcement bar in the concrete) leading to Equation 9 and generalising Equation 5:

$$P(y, e) = P(y|e) \cdot P(e)$$

(9)

Where $P(y,e)$ is the joint distribution and $P(y|e)$ is the conditional distribution of $y$ knowing the value $e$.

The average ion concentration is obtained by summing the ion concentration over the entire space field (first term in Equation 10). This double integration is simplified by considering the local average of $y$ for a known position $e$ (second term in Equation 10). The result thus obtained corresponds to a weighted mean over the space field (last term in Equation 10):

$$E(y) = \int \int y \cdot dP(y, e) = \left( \int y \cdot dP(y|e) \right) \cdot dP(e) = \int E(y|e) \cdot dP(e)$$

(10)

Similar to the average, $E(y)$ or $\mu_y$, the other parameters of the normal or log-normal distributions ($\sigma_y$, $\lambda_y$ and $\xi_y$) fitted to the simulation results can be calculated through Equation (10) (Fig. 2).

The normal or log-normal probability distributions extend on an infinite field and may not be representative of reality. For example the steel rebar is always in concrete and thus the concrete cover $e$ can not be negative, i.e. outside the concrete. In all cases considered here, the normal and log-normal probabilistic distributions are truncated at the concrete surface, $e=0$.

4 MODELLING OF CHLORIDE ION TRANSPORT

The chloride ion transport into the concrete cover is modelled using a numerical model called “TransChlor” (Conciatori, 2005). The relevant parameters of this model are the microclimate, the presence of de-icing salts and the permeability of the concrete structural element. The microclimate is a function of the structural element’s exposure to solar radiation (zones in the shade are distinguished from those exposed to the sun), the annual average carbon dioxide concentration and the geographically linked weather conditions (air temperature, relative humidity and precipitation).

“TransChlor” considers the thermal diffusion process and the hydrous transport by capillarity and vapor diffusion as a function of the carbonation state in order to simulate the chloride ion transport in concrete (Fig. 3) (Conciatori, et al., 2002; Denarié, et al., 2003; Conciatori, 2005). Hydrous transport parameters include the capillarity coefficient $D_{cap}$, the vapor diffusion coefficient $D_v$ and the chloride ion diffusion coefficient in water $D_{Cl}$. Chloride ion convection is treated by means of a probability distribution based in turn on the hydrous transport probability distributions.

The various transport processes are modelled by employing Fick’s Equations (11 to 14), and using the finite element method to solve for the ion propagation within the concrete and the finite differences method (implicit method) to solve the progression in time of the propagation front.
The thermal diffusion process (Equation 11) takes into account the specific heat capacity $c_T$ and thermal conductivity $\lambda_T$. The specific heat is obtained by summing the specific heat of each concrete component with respect to the volume, i.e. aggregates, pore water, cement paste, etc. Thermal conductivity varies according to the thermal energy $T$ stored in the concrete and the concrete water content $w$ (Hamfler, 1988).

Equation (12) for hydrous transport considers the contact with water vapor diffusion $D_h$ and liquid water capillary $D_{cap}$ coefficients. Vapor transport is a function of temperature $T$ and the concrete water content $w$ (Bazant and Najjar, 1971). The temperature effect is described using the Arrhenius law (Conciatori, 2005). Liquid water transport is a function of the concrete temperature $T$, the water content $w$, the relative concrete humidity $h_r$ and the concrete contact time with liquid water $t_{contact}$.

\[
\frac{\partial T}{\partial t} = \text{div} \left( \frac{\lambda_T(T, w)}{c_T(w)} \text{grad}(T) \right) \tag{11}
\]

\[
\frac{\partial h_r}{\partial t} = \text{div} \left( D_h(T, h_r) \cdot \text{grad}(h_r) \right) - D_{cap}(t_{contact}, h_r, E/C, T) \cdot \text{grad}(h_r) \tag{12}
\]

The carbon dioxide $\text{CO}_2$ transport documents the carbonation progression in concrete. The extent of carbonated concrete is required to determine the amount of chloride ions absorbed by the cement paste. Concrete carbonation depth $x_c$ (Equation 13) is obtained by considering the water amount, the concrete permeability to $\text{CO}_2$ and the chemical carbonation reaction rate (Papadakis et al., 1990). The external conditions are represented by the molar carbon dioxide concentration $[\text{CO}_2]$. The carbonation reaction rate is contained by the calcium hydroxide $[\text{Ca(OH)}_2]$ and calcium silicate hydrate $[\text{CSH}]$ molar concentrations. Finally, the water amount and the concrete permeability are considered by means of the concrete carbon dioxide diffusion coefficient $D_{c,\text{CO}_2}$.

\[
x_c = \sqrt{\frac{2 \cdot [\text{CO}_2] \cdot D_{c,\text{CO}_2}}{[\text{Ca(OH)}_2] + 3 \cdot [\text{CSH}]}} \cdot \sqrt{t} \tag{13}
\]

\[
\frac{\partial C}{\partial t} = \text{div} \left( R_{Cl} \cdot c_f \cdot D_h \cdot \text{grad}(h_r) + w(h_r, T) \cdot D_{Cl} \cdot \text{grad}(C) \right) + R_{Cl} \cdot c_f \cdot \left( D_{cap} \cdot \text{grad}(h_r) \right) \tag{14}
\]

\[
C = C_f + C_b = c_f \cdot w + c_f^\beta \cdot \gamma \tag{15}
\]

The transport of chloride ions (Equation 14) is a function of the chloride ion diffusion through the pore water $w$ and the movement of entrained chloride ions dissolved in the water moving through the concrete (Equation 12) (Conciatori, 2005), also called convection. The retardation of the chloride ion front with respect to the convection induced water movement is taken into account by including the retardation coefficient $R_{Cl}$ (Lunk et al., 1998). Hardening cement paste and the carbonation rate facilitates the binding of a certain amount of chloride ions (Lindvall, 2001; Ritthichaumy et al., 2002; Sugiyama et al., 2003; Tang, 1996; Tang and Nilsson, 1996). Thus, chloride ions bound in the cement paste $C_b$ are distinguished from the free chloride ions moving through the concrete $C_f$. The total chloride content in the concrete $C$ is related at the Freundlich isotherm $\beta$ and $\gamma$ using concentration of free chloride in water $c_f$ (Equation 15). Chloride ion diffusion coefficient in water $D_{Cl}$ varies according to the temperature, but due to the lack of test results, only the isothermic diffusion coefficient for 20°C is used.
The complexity of the physical and chemical phenomena is evident in the modelling system’s non-linearity. The modelling system is iteratively solved since an analytical solution of the probabilistic approach is complex and time intensive (Conciatori, 2005).

5 PROBABILITY OF CORROSION INITIATION

The probabilistic transport parameters \(D_{cap}, D_h\) and \(D_{cl}\) (see Fig. 3) are introduced into the “TransChlor” model presented in chapter 4 by means of the Rosenblueth method described in Ch. 2. These parameters can be modelled by independent random variables. Since we have three parameters, the Rosenblueth method requires \(2^3=8\) runs of the numerical code. The eight results are combined to obtain the distribution of the chloride ion concentration (see chapter 2.3). In the following (chapter 6), this distribution is assumed to be log-normal. From this result, the corrosion loading \(L\) is obtained in terms of probabilistic ion concentration at the steel reinforcement by integrating the ion density with the concrete cover variability (see chapter 3). The steel reinforcement corrosion resistance \(R\) is represented by critical chloride ion content \(c_{cr}\) required to initiate corrosion.

The probability of corrosion initiation is obtained by comparing the corrosion loading \(L\) to the steel reinforcement corrosion resistance \(R\), producing the limit function \(G = R - L\) (Spiegel, 1981; Gehlen and Schiessl, 1999). From this, the initiation probability is derived:

\[
P_f = P(G < 0) = P(R - L < 0) = \int P(R < L|L = x) \cdot P_L(x) \cdot dx = \int P(R < x) \cdot P_L(x) \cdot dx
\]  

(16)

if the random variables \(R\) and \(L\) are independent. To compute the corrosion initiation probability \(P_f\) we use the last term in Equation 16, the convolution integral of the probabilities that \(L\) is equal to \(x\) and that \(R\) is lower than \(x\) (Spiegel, 1981; Kraker et al., 1982; Lindvall, 2001). Finally we have.

\[
P_f = \int_{-\infty}^{\infty} f_L(x) \cdot F_R(x) \cdot dx
\]  

(17)

where \(f_L\) denotes the density of \(L\) and \(F_R\) the cumulative distribution of \(R\). The corrosion initiation probability is then compared to an acceptable target value determined through considerations for the given structural element (Conciatori, 2005). The corrosion initiation probability is associated with the incurred risk of deterioration. The owners typically manage interventions on structures on the deterioration. Our risk analysis method thus provides an informative tool for the dialogue between owner and structural engineering specialists (Conciatori, 2005). In short, our evaluation provides a decision basis for maintenance interventions.

6 NUMERICAL EXAMPLES

De-icing salts are spread on roadways in winter periods to remove snow and ice. In this way, chloride ions come into contact with concrete structural elements through vapor by salt-laden mist or through liquid water by direct contact or splashed on structural elements by passing vehicles. The “TransChlor” model boundary parameters are the microclimate, the presence and concentration of de-icing salts (Conciatori, 2005):
- The microclimate depends on the geographical location and takes into account air temperature, air relative humidity and precipitations. The corresponding data has been obtained from different meteorological stations in Switzerland.
- The element exposure to the road traffic is stagnant water, splash water and mist, but only the two last exposures are addressed in these examples.
- The concentration of de-icing salts evolves in time according to the number of salt spreader interventions and according to local weather conditions for the given geographical location. The chloride ion concentration depends primarily on the type of salt spreader intervention, i.e., mechanical spreading in the solid or brine form or automatic brine spreading assisted by a monitoring of a road section.

The “TransChlor” model uses a finite element algorithm and finite differences to solve the system of equations (Equations 11-15). The examples use unidirectional analysis, with finite element size of one millimetre. The structure element is represented by a thickness of 100 millimetres. The time step choice of one hour is well adapted to the boundary parameters. This time step decreases when the boundary parameters change drastically. This decrease is studied with regard to convergence of the method (Conciatori, 2005).

Thermal transfer validation was carried out by comparing temperatures measured in laboratory samples and the “TransChlor” simulations results. The various temperature levels were between 20°C to -20°C (Conciatori, 2005). Liquid water transport was validated using laboratory data from testing by means of sample immersion in brine at low temperature (Conciatori et al., 2005; Conciatori and Brühwiler, 2006). Other cases of capillary transfer (Conciatori et al., 2005) were also simulated with the “TransChlor” model.

Chloride ingress was reproduced accurately for the case of chloride convection by water. The experimental data was obtained from a chloride sensor (consisting of an optical fiber) placed in a concrete sample (at 18 mm from the surface) that was subjected to a capillary test. The chloride and water front passage is shown after about 19 hours, and the simulation after about 33 hours. After some differences in the first days of exposure, the measurements and simulations were identical after 8 days (Conciatori et al., 2005).

6.1 Example 1

A comparison between different concrete cover thicknesses is developed in this first example. Two different meteorological stations are considered. On one hand, the Davos meteorological station (mountainous area in Switzerland, 1600 m above sea level) and on the other hand the Pully meteorological station (moderate climate of the Swiss Plateau boarding Lake Geneva, 400 m above sea level) (Conciatori, 2005).

Parameter variation is modelled by normal and log-normal probability distributions with the values given in Table 3. The vapor water diffusion coefficient $D_h$ and the capillarity coefficient $D_{cap}$ values have been determined from laboratory testing (Conciatori, et al., 2006). The chloride ion diffusion coefficient in water $D_{Cl}$ is used in this example as a deterministic parameter. The steel corrosion resistance is expressed in terms of the critical chloride ion content $c_{cr}$, the chloride concentration required to initiate corrosion. The values given in Table 3 are valid for ordinary steel in non-carbonated concrete (Schiegg, 2002; Conciatori, 2005). All random parameters were obtained from experimental data using the method of moments or maximum likelihood.

Table 3: Probability distribution parameters for Example 1.

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\mu$ or $\lambda$</th>
<th>$s$ or $\xi$</th>
<th>Distribution</th>
</tr>
</thead>
</table>

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In the case of splash exposure (Fig. 4 a and c), the free chloride ion concentration does not vary significantly. With a concrete cover thickness of 15 millimetres, for example, the probability of corrosion initiation increases rapidly and reaches 20 % after during the first year of exposure. Irrespective of concrete cover, chloride ions penetrate very rapidly. Chloride ions accumulate in the concrete cover up to a depth of 20 to 60 mm and then migrate further into the concrete by diffusion. The rapid chloride ion movement through the transition zone, i.e. the first 10 to 20 mm of the concrete cover, may also have the effect that the concentration of chloride ions could decrease in this transition zone during the summer periods by washing out and drying. Consequently the corrosion initiation probability could be smaller in the transition zone than in the accumulation zone, i.e. in 20 to 60 mm depth of the concrete cover.

When the structural element is exposed to a salt-laden mist (Fig. 4 b and d), the corrosion initiation probability and thus the corrosion initiation time increases significantly with increasing concrete cover. In this case, chloride ion transport is predominantly achieved by vapor water diffusion.

The concrete exposure type is of great importance, for each exposure type induces different chloride ion propagation processes. Splash exposure leads to rapid chloride ion transport by convection, while mist exposure induces predominately chloride ion diffusion. This produces corrosion initiation times differing by ten to a hundred fold.

6.2 Example 2

A comparison between different concrete permeability is developed in this second example. Climatic data is taken from the meteorological station same as the example 1. Parameter variation is modelled, as in Example 1, by normal and log-normal probability distributions assuming the values given in Table 4. In this example, three different concrete permeabilities are considered, i.e., Concrete A with low permeability, Concrete B with medium permeability and Concrete C with high permeability.

Table 4: Probability based parameters for Example 2.

<table>
<thead>
<tr>
<th>Probability distribution parameters</th>
<th>Concrete A</th>
<th>Concrete B</th>
<th>Concrete C</th>
<th>Probability distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$ or $\lambda$</td>
<td>$\sigma$ or $\xi$</td>
<td>$\mu$ or $\lambda$</td>
<td>$\sigma$ or $\xi$</td>
</tr>
<tr>
<td>$D_h$ [mm$^2$/s]</td>
<td>-9.937</td>
<td>0.657</td>
<td>-9.164</td>
<td>0.657</td>
</tr>
</tbody>
</table>
Given a splash exposure, (Fig. 5 a and c), the concrete permeability does not significantly influence the chloride ion penetration, and very rapidly, i.e. after 5 years, a very high corrosion initiation probability is reached indicating this reinforced concrete member’s durability is insufficient.

For a salt-laden mist exposure, (Fig. 5 b and d), the corrosion initiation probability increases and thus the corrosion initiation time decreases with increasing concrete permeability. This is explained by the relatively slow process of transferring chloride ions dissolved in water vapor.

### 7 CONCLUSIONS

Complex processes such as chloride ion penetration into concrete resulting in steel reinforcement corrosion, involving significant parameter variability is modelled using the Rosenblueth method. This method realistically predicts the evolution of chloride ingress into concrete to determine in a second step corrosion initiation probability using a complex physical model combined with a simple probabilistic approach. This is important since nowadays probabilistic analyses are often provided using simplified chloride transport modelling combined with numerically heavy iterations, which would be impossible to perform with complex but realistic transport model.

Yet, a comparison with Monte Carlo method is necessary to fully validate the use of Rosenblueth method. Current preliminary studies regarding this comparison show promising results.

The corrosion initiation probability is obtained by using an original transport model which considers the relevant parameters including the exposure type, the concrete permeability, the concrete cover, and the critical corrosion initiation chloride ion concentration. These parameters are modelled using normal and log-normal probability distributions.

The two numerical examples indicate that (1) for splash exposure, significant corrosion initiation is reached after a relatively short time, i.e. up to 5 years, irrespective of the concrete cover or the concrete permeability, and (2) for mist exposure, these parameters are significant producing a corrosion initiation range extending from 5 to more than 50 years depending on the parameters.
8 NOTATIONS

\( \beta \) skewness Pearson measure,
Freundlich isotherm parameter,

\( C \) total chloride ion concentration with respect to the concrete volume, (kg/m\(^3\))

\( C_B \) bound chloride ion concentration with respect to the concrete volume, (kg/m\(^3\))

\( C_F \) free chloride ion concentration with respect to the concrete volume, (kg/m\(^3\))

\([\text{Ca(OH)}_2]\) portlandite carbonated molar concentration, (mol/m\(^3\) air)

\( c_{ct} \) threshold chloride ion content, (%cement mass)

\([\text{CO}_2]\) carbon dioxide molar concentration, (mol/m\(^3\) air)

\( c_f \) free chloride ion concentration moving through the concrete interstices with respect to the solution volume, (kg/m\(^3\))

\( \text{cov} \) variation coefficient,

\([\text{CSH}]\) calcium silicate hydrate molar concentration, (mol/m\(^3\) concrete)

\( c_T \) heat capacity with respect to one cubic meter of concrete, (kJ/(m\(^3\).K))

\( D_{\text{cap}} \) capillarity coefficient, (mm/s)

\( D_{Cl} \) total chloride ion diffusion coefficient, (mm\(^2\)/s)

\( D_{c,CO_2} \) concrete carbon dioxide diffusion coefficient, (mm\(^2\)/s)

\( D_h \) vapor water diffusion coefficient, (mm\(^2\)/s)

\( e \) concrete cover, (mm)

\( \text{erf} \) error function,

\( E[x], \mu_x \) sample weighted mean of \( x \),

\( E(\mu_x^3) \) skewness of \( x \),

\( F(X_1,X_2) \) two variable joint distribution,

\( F(X_1), F(X_2) \) distributions of two parameters \( X_1 \) and \( X_2 \),

\( f(\mu,\sigma) \) function of a continue normal probabilistic law,

\( f(\lambda,\xi) \) function of a continue log-normal probabilistic law,

\( F_1,F_2 \) distribution associated with a probability parameter,

\( f_l \) continuous probability function, representing load,

\( F_R \) cumulated continuous probability function, representing resistance,

\( G \) limit function,

\( \gamma \) Freundlich isotherm parameter,

\( h_r \) relative humidity in the concrete pores, (-)
i  probability parameter number (1,n),
j  point or probability parameter number,
k  simulation sample position (1,2^n),
ζ  log-normal distribution parameter, corresponding to the sample Napierian logarithms standard deviation,
λ  log-normal distribution parameter, corresponding to the sample Napierian logarithms average,
L  chloride ion action at the reinforcing steel level,
λ_P    concrete thermal conductivity, (W/(m.K))
m  total number of simulations with the Rosenblaueth method,
n  number of parameters modelled by a probability distribution,
μ  normal law parameter, representing the average,
P  random operator,
P_f   corrosion initiation probability,
Q_k  probabilities associated combination F_{ij} of variables i.
R  material corrosion resistance,
R_{Cl}  chloride ion front retardation coefficient,
σ  normal distribution parameter, representing the standard deviation,
α_m  log-normal distribution parameter,
α_p  log-normal distribution parameter,
s  parameter representing the positive integer,
S_x  surface under a probabilistic distribution,
t  time, (s)
t_{contact}  concrete contact time t with liquid water,
T  temperature, (°C)
var[x], σ^2  sample variance x,
w  concrete water content, (kg/m³ concrete) or dimensionless unit
x_c  carbonation depth in the concrete cover, (mm)
x_{ij}, X_1, X_2  probabilistic parameter points,
y_k  simulation results.

9 REFERENCES


Pearson, E. S., Johnson, N. L. & Burr, I. W. (1979), Comparisons of the percentage points of distributions with the same first four moments, chosen from eight different systems of frequency curves, *Communication in Statistics, 8*(3), 191-229.


Fig. 1: The drawing illustrates how the continuous distribution on the left can be approximated by the discrete law with two support points shown in the middle. If the weight in a beam is distributed according to the continuous distribution and supported at the two support points with forces $F_1$ and $F_2$, it is in balance.
Fig. 2: Chloride ion concentration as a function of the concrete cover.
Fig. 3: Transport model "TransChlor" (schematic distributions).
Fig. 4: Corrosion initiation probability as a function of concrete cover thickness, a) Davos: splash exposure, b) Davos: mist exposure, c) Pully: splash exposure, d) Pully: mist exposure.
Fig. 5: Corrosion initiation probability as a function of concrete permeability in Davos, a) splash exposure, b) mist exposure, and in Pully c) splash exposure, d) mist exposure.