Abstract—We theoretically compare the performance of optical discrete multi-tone (DMT) and pulse amplitude modulation (PAM) using intensity modulated, direct detection (IMDD). PAM is a lower cost, lower complexity solution than DMT, however it is more vulnerable to chromatic dispersion on the C band. We compare DMT and PAM taking into consideration the interplay of laser linewidth, fiber length, transmission rate, and channel bandwidth. We use a semi-analytical model to examine bit error rates. We study how system parameters shift the performance advantages between DMT and PAM. Our model can also be used to find the best hardware solution and frequency band for a target modulation format and bit error rate.

Index Terms—PAM, DMT, C-band, O-band, laser linewidth, chromatic dispersion, bandwidth.

I. INTRODUCTION

HIGH demand for expanded network capacity drives new standardization into 400 Gb/s optical fiber transmission. The main focus of this paper is on short haul applications targeting these speeds, such as data centers (≤10 km) and passive optical networks (≤60 km). Pulse amplitude modulation (PAM) and discrete multi-tone (DMT) are both compatible with cost-effective direct detection, and are popular choices to reach 400 Gb/s.

While various experimental demonstrations have witnessed PAM or DMT perform best, we probe the origin of the variation in reported trials. For example, experimental demonstrations of 112 Gb/s PAM transmission using a SiP modulator [2], 100 Gb/s Nyquist PAM-4 transmission using an electro-absorptive modulated laser have appeared, both on the O-band (1310 nm) [3]. In addition, there have been reports on 128 Gb/s PAM-4 transmission system using a multi-electrode silicon photonic Mach Zehnder modulator [4], EML-based 4 lanes of 112.5 Gb/s PAM4 [5], and 112 Gb/s PAM4 Amplifier-free using O-band DML [6]. There are also several reports of DMT transmission over 100 Gb/s, such as our previous work with SiP modulators, where we reached 120 Gb/s on the O-band [7], another SiP modulator achieving 130 Gb/s on C-band (1550 nm) [8], beyond 100 Gb/s transmission of SSB-DMT on O-band [9], 4channel of 100 Gb/s DMT using O band silicon photonic modulator [10], and 100 Gb/s dual side-band DMT with dispersion compensation on C-band [11].There has also been a hybrid PAM/DMT demonstration at 112 Gb/s using a directly modulated laser on O-band [12].

Several comparisons have been made between PAM and DMT, both experimentally and theoretically. In the experimental comparison of PAM versus DMT using an O-band directly modulated laser [13], DMT performed better than PAM at 2.2 km. In the analytical comparison between PAM and DMT [14], we used the same model for PAM as in this paper, but our analysis was for double sideband DMT on the O-band. The main degradation sources for DMT are assumed to be receiver noise and clipping noise. Another example is an experimental comparison between PAM and DMT on O-band up to 10 km propagation distance in [15]. In this paper the author compared the implementation complexity and showed that pulsed shaped PAM requires larger number of complex multipliers to be implemented. Unlike previous comparisons we focus on SSB-DMT on C-band and non-pulse shaped PAM on O-band. Our analysis covers up to 100 km, therefore we can neglect fiber nonlinearity.

We can generate single side-band DMT (SSB-DMT) at the transmitter using dual drive Mach
Zehnder modulators (DDMZM) [16]. This complex signal contains the same data as a DSB-DMT signal made real with Hermitian symmetry, but occupies half the optical spectrum. We already require a digital-to-analog (DAC) converter for DMT, and the DAC can be exploited for SSB. Note that while the optical spectral efficiency is increased by two compared to DSB-DMT, the required DAC bandwidth (or electrical spectral efficiency) is unchanged. The DAC hardware and transmitter DSP is therefore the same for either DSB-DMT or SSB-DMT implementations. PAM, on the other hand, does not require a DAC; PAM would lose much of its low complexity advantage in adopting an SSB implementation. Attenuation on O-band is greater than that on C-band, but on C-band the chromatic dispersion is nonzero. Power fading is the most important degradation caused by chromatic dispersion on single mode fiber (SMF). Therefore, in our comparison we assume that SSB is an option available for DMT and not PAM. In other words, our assumption will lead to PAM suffering from power fading, but not DMT.

Signal to signal beating interference (SSBI) is present in both O-band and C-band and is a major source of impairment for DMT. For O-band modulation, SSBI for SSB-DMT can be mitigated, while on C-band it remains a factor [17]. Therefore, there is a trade-off between attenuation on O-band and dispersion on C-band in comparing DMT with PAM. Depending on the fiber lengths involved, either SSB-DMT on C-band or PAM on O-band may be preferred.

Our analytical study of SSB-DMT on C-band in [17] shows the effect of phase to amplitude noise (P2A), inter-carrier interference (ICI), SSBI, attenuation C-band. Using this analysis we also optimized SSB-DMT in terms of signal-to-carrier power ratio to achieve highest possible throughput in any system condition. In this paper we use the same analysis for SSB-DMT.

We concede that PAM will typically outperform DMT on the O-band; due to the lower cost of PAM equipment, it is a better choice on O-band. Similarly, DMT will typically outperform PAM on the C-band as overcoming power fading on PAM will lead to excess complexity. Therefore, we focus on the detailed performance comparison of PAM on O-band and DMT on C-band.

In section II we provide a mathematical model for SSB-DMT on C-band and PAM on O-band. Section III introduces equations to calculate bit error rate (BER) for PAM and DMT modulation formats, relying on methodology from [17] to optimize DMT performance. Using results from section III, in section IV we expand on [1] to examine the interplay of laser linewidth, fiber length, transmission rate, and channel bandwidth. We offer some concluding remarks in section V.

II. SYSTEM MODEL

A. DMT

In this paper we will examine DMT performance on the C-band. We model the SSB-DMT launched signal as in our previous work [17]. The DMT signal in the time domain, $s_{\text{DMT}}(t)$, equals

$$A_c e^{j 2 \pi f_c t + j \Phi(t)} \left(1 + \gamma \sum_{k=1}^{N} p_k d_k e^{j 2 \pi k \Delta f t}\right),$$

where $A_c$ is the carrier amplitude, $f_c$ is the carrier frequency, $\gamma^2$ is the signal-to-carrier power ratio (SCR), $p_k$ and $d_k$ are the allocated power and normalized complex amplitude of the $k$th subchannel, respectively, $N$ is the number of subchannels, and $\Delta f$ is the subchannel frequency spacing. Laser phase noise, $\Phi(t)$, with linewidth of $\Delta \nu$, is modeled by a Wiener process.

Taking into account the chromatic dispersion of fiber on C-band and the laser phase noise, the received signal after photo detection is

$$r(t) = 2P_c Re \left[ \gamma \sum_{k=1}^{N} p_k d_k e^{j 2\pi k \Delta f t + j \rho_k(t) + j \theta_k} \right] + \left[ P_c \gamma^2 \sum_{k=1}^{N} |p_k \cdot d_k|^2 \right] + P2A + SSBI,$$

where $\rho_k$ is the phase fluctuation of the $k$th subchannel, $P_c$ is carrier power, and $\theta_k = -2 \pi k \Delta f T_k$, where $T_k$ is the time delay or the walk-off for the $k$th subchannel. The first term in (2) is the DMT signal distorted by inter-carrier interference (ICI), random phase rotation (PR), and power degradation. The second term is DC offset which will not affect signal quality. The P2A is the combination effect of chromatic dispersion and phase noise on carrier itself, this effect translates carrier phase noise into amplitude noise.
Summarizing our results from [17], the power of P2A noise per subchannel is calculated by multiplying carrier power by the in-band subchannel P2A power, i.e.,

\[
\sigma^2_{k(P2A)} = P_c \int_{(k-1)\Delta f}^{k\Delta f} PSD_{P2A}(f) df,
\]

(3)

where \(PSD_{P2A}(f)\) is the power spectral density for P2A noise from [18]. The power of the SSBI noise per subchannel is calculated from

\[
\sigma^2_{n(S)} = \frac{P_s}{N} = \frac{\gamma^2 P_c}{N},
\]

(4)

the signal power in the nth subchannel. We assume all subchannels are loaded with data and there is no zero padding. This yields a signal-to-signal beat noise with SNR of

\[
\sigma^2_{k(SSBI)} = \frac{1}{P_c} \sum_{n=-N}^{N} \sigma^2_{n(S)} \sigma^2_{k-n(S)},
\]

(5)

The ICI power per subchannel is calculated by studying the interaction of dispersion with phase noise, yielding

\[
\sigma^2_{k(ICI)} \approx \beta_k \frac{N^2}{N^2} \left( N^2 + \frac{1}{3} M_k^2 - N M_k - \frac{1}{3} \right),
\]

(6)

where \(\beta_k = 2\pi \Delta \nu T_k\) is the phase noise power in the \(k\)th subchannel, \(T_k\) is the sampling interval of the DAC and \(M_k = T_k / T_s\) is the delay relative to the carrier (in number of samples) for the \(k\)th subchannel. The full study of P2A and SSBI is provided in [17].

The signal-to-noise ratio (SNR) per subchannel for each term in (1) is calculated separately. Transmitter and/or receiver bandwidth limitations are examined by assuming the overall channel frequency response has a Gaussian shape parameterized by its 3 dB bandwidth. The receiver noise is assumed to be additive white Gaussian noise (AWGN).

Taking the SNR per subchannel, we scale appropriately to have an overall SNR of 22 dB when only receiver noise is present, no other impairments. This same value of SNR is used for PAM and is selected as it represents acceptable performance over a wide range of typical system parameters such as the ratio of bit rate to bandwidth, and the laser linewidth.

### B. O-band PAM

For PAM modulation we assume a rectangular pulse shape \(p(t)\), and a Gaussian shaped channel impulse response \(h(t)\) whose bandwidth is parameterized by its 3 dB bandwidth (as with DMT analysis). The received PAM signal is the time domain, \(s_{PAM}(t)\), equals

\[
A_c e^{2\pi f_c t} \left( 1 + \sum_{n} a_n g(t - nT_{sym}) \right)
\]

(7)

where \(g(t) = p(t) * h(t)\), and \(a_n\) is the \(n\)th PAM symbol, and \(T_{sym}\) is the symbol time. Note that \(\{a_n\}\) are scaled to achieve unity power in the multilevel signal. The only distortion on PAM is due to the bandwidth limitation, and the only noise is receiver thermal noise which modeled as additive white Gaussian noise with SNR of 22 dB. Attenuation will cause the SNR to decrease with increasing fiber length. To mitigate inter-symbol interference (ISI) caused by a band-limited channel, we assume a proper equalization technique; the overall effect of a band-limited channel with an equalizer will be calculated in the next section.

### III. BER CALCULATION

In this section at first we define an approach that provides a fair comparison between PAM and SSB-DMT. The second subsection covers BER calculation for SSB-DMT exploiting the knowledge of SNR per subchannel from [17]. Finally BER for O-band PAM with a decision feedback equalizer is calculated.

#### A. Comparison Framework

To justly compare PAM and DMT, we assume the total signal power \((P_s)\) and the receiver noise power spectral density \((N_0)\) for both cases are the same. Equivalently, the ratio of energy per symbol, \(E_s\), to \(N_0\) is the same for both PAM and DMT. The overall DAC bandwidth, \(BW_{DAC}\), will be exploited by SSB-DMT. For DMT, the receiver noise induced SNR, \(SNR_{RX}\), is

\[
SNR_{RX} = \frac{P_s}{P_N} = \frac{E_s \times R_s}{N_0 \times BW_{DAC}}.
\]

(8)

We can write the ratio \(E_s / N_0\) in terms of the the receiver noise induced SNR as

\[
\frac{E_s}{N_0} = SNR_{RX} \times \frac{BW_{DAC}}{R_s}
\]

(9)
The SNR for DMT, excluding all noises other than receiver thermal noise, is by definition $SNR_{RX}$ since it is referenced to the total DAC bandwidth. For the case of PAM with baud rate $R_s$, the noise equivalent bandwidth is approximately $2R_s$ and the out-of-band noise is removed by a low pass filter at the receiver. While the thermal noise contribution will be different for PAM, $E_s/N_0$ will be the same. We use $BW_{DAC} = 32$ GHz as a reference value for our calculations.

### B. C-band SSB-DMT

The SNR per subchannel is found from contributions identified in section II.A and is given by

$$
\frac{1}{SNR_k} = \frac{1}{SNR_{k(ICl)}} + \frac{1}{SNR_{k(P2A)}} + \frac{1}{SNR_{k(SSBI)}} + \frac{1}{SNR_{(RX)}},
$$

(10)

where $SNR_{k(ICl)}$, $SNR_{k(P2A)}$, and $SNR_{k(SSBI)}$ are provided in section III of [17]. We model the bandlimited channel frequency response as a Gaussian low pass filter with 3 dB bandwidth $B$. We recalculate the SNR per subchannel taking into account the attenuation induced by the channel frequency response. Knowing the SNR per subchannel and target bit rate, we use Chow’s water-filling technique [19] to find the number of bits allocated to each subchannel, as well as the power allocation for each subchannel. Using the power allocation for each subchannel, the resultant received SNR for each subchannel is found. We calculate BER for each subchannel (theoretical QAM BER for the assigned bit allocation) by assuming Gaussian noise. Details of these manipulations are available in [17], including the determination of the optimal signal to carrier ratio for DMT. Note that the power allocation and signal to carrier ratio does not change the overall signal power, only its distribution across frequency.

### C. O-band PAM

Because we are considering PAM on O-band, the only significant degradation is fiber attenuation. The BER of PAM with modulation order $M$ in AWGN is

$$
BER_{PAM} = \frac{M - 1}{M \log_2 M} erfc\left(\sqrt{\frac{3SNR}{M^2 - 1}}\right),
$$

(11)

where $erfc$ is the complementary error function. We next find the SNR value to include in this expression when referencing the receiver noise power to that used for DMT.

In the case of a non-flat channel frequency response, the PAM signal experiences ISI; to reduce this effect an equalizer is used. Nonlinear decision-feedback equalizers have been shown to outperform linear feed-forward equalizers, hence we focus on them. PAM BER performance with a nonlinear decision-feedback equalizer was found in [14] via a Gaussian noise approximation. The SNR is reduced due to ISI/equalization. We use this approach and write the cumulative SNR at the equalizer output as

$$
SNR_{DFE} = 2E_s\exp\left\{\frac{1}{f_s} \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} \ln|Y(f)| \, df\right\} =
$$

$$
\frac{2SNR_{RX}BW_{DAC}}{R_s}\exp\left\{\frac{1}{f_s} \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} \ln|Y(f)| \, df\right\},
$$

(12)

where $f_s$ is the sampling frequency, $R_b$ is bit rate, $R_s = R_b/\log_2 M$ and

$$
Y(f) = \sum_{n=-\infty}^{\infty} |S(f - nf_s)H(f - nf_s)|^2,
$$

(13)

where $S(f)$ is the transmitted PAM spectrum and $H(f)$ is the channel frequency response. We assume the rectangular PAM pulse sees the same Gaussian channel used in the DMT performance calculation. BER is calculated from $SNR_{DFE}$ and (11).

### IV. Performance Comparison

We compare PAM4 and SSB-DMT under a variety of operational constraints, examining the triplet ($L$, $\Delta
nu$, $B$), as well as the bit rate. Our goal is to identify under which conditions SSB-DMT on C-band or PAM4 on O-band would be a better modulation choice. For SSB-DMT, some cases may have unattainable target BER due to a limited power budget. We are careful to consider the same power budget for all cases of SSB-DMT and PAM4. Note that while DSP can be used to mitigate SSBI, see for example [20], we consider only standard DMT.
DSP, as in [13]. The forward error correction is not implemented in the DSP and its threshold in all comparisons is $3.8 \times 10^{-3}$. Note that in all cases in this section DMT refers to SSB-DMT.

A. Impact of Bandwidth Constraints

In this section we use the analytical tools developed earlier to examine the impact of bandwidth constraints imposed for different hardware solutions. For instance, the use of low-cost silicon photonic modulators can become a bottleneck in the overall channel bandwidth. In one set of comparisons we vary both bit rate and channel bandwidth simultaneously, i.e., where a choice of modulators might exist. In another set of comparisons, we assume a fixed hardware solution limiting channel bandwidth to 25 GHz, and we vary the bit rate. In either case, we present results as a function of the ratio of bit rate to system bandwidth. A lower ratio corresponds to less aggressive bit rates for a given hardware solution, while a higher ratio corresponds to aggressively pushing high bit rates through a restricted bandwidth.

1) Fixed Bandwidth: Figure 1 examines performance when the bandwidth is fixed to 25 GHz (e.g., a given modulator), laser linewidth is equal to 500 kHz, and the bit rate ranges from 50 to 150 Gb/s. For easy comparison with later results, we label the $x$-axis with the ratio of bit rate to system bandwidth. As PAM4 performance is most influenced by fiber length, we limit our examination to three cases, $L = 10, 20$ and 30 km.

As fiber length is changed, PAM4 and DMT performance varies as seen in Figs. 1a, b, and c. For PAM4 this is a simple effect of attenuation with length. For DMT length contributes to several terms in the SNR per subcarrier, leading to a more complex effect on performance. In addition, DMT performance changes markedly as we change bit rates. For the shortest fiber (Fig. 1a), PAM4 has a clear advantage due to the negligible attenuation. By 30 km (Fig. 1c), the lower attenuation on C-band than O-band has tipped the scales to give DMT an edge at all swept bit rates. At a length between these extremes, Fig. 1b, the performance of PAM is almost the same as DMT at all bit rates considered.

2) Bit rate and bandwidth varying: When increasing the baud rate, the PAM noise-equivalent bandwidth increases, and overall noise power increases. This effect worsens the BER of PAM as we increase baudrate (equivalently bit rate for a given $M$) for a fixed spectral density. As expected, the higher the ratio of bit rate to system bandwidth, the greater the distortion and the worse the BER for PAM in Fig. 2. Furthermore, the BER degrades as the bit rate increases from 80 Gb/s in Fig. 2a to 120 Gb/s in Fig. 2c.

In the case of DMT, greater bit rates require higher modulation levels, i.e., a constellation with more bits per symbol, but requiring greater SNR;
as with PAM, performance varies not only with the ratio but also the absolute bit rate. BER performance degrades as we push bit rate (the three curves are progressively higher). For 80 Gb/s (Fig. 2a) the performance of DMT is always better than PAM4, because for low bit rates the water-filling technique (Chow’s algorithm) leads to robust, lower QAM orders. By increasing bit rate to 100 Gb/s (Fig. 2b), PAM4 and DMT BER curves intersect; PAM4 outperforms DMT in lower bit rate to bandwidth ratios. Further increasing bit rate (Fig. 2c) moves the intersection to higher ratios.

B. Impact of Fiber length and Linewidth

1) Fiber length: Let us focus on fiber length in Fig. 3, for fixed system with bit rate (100 Gb/s), linewidth (100 kHz) and channel bandwidth (25 GHz). Our tools capture the complex impact of fiber length on DMT (in P2A, ICI, PR and power degradation) and quantify at which length DMT and PAM4 performance crossover. Comparisons were also done for 80 and 120 Gb/s (not included in this paper) and confirm that high attenuation in O-band eventually leads to PAM4 performance worse than DMT; that crossover propagation distance varies with bit rate.

2) Joint Impact of Linewidth and Fiber Length: While PAM4 is unaffected by laser linewidth, DMT performance will be degraded with wider linewidth. The previous comparisons assumed a narrow (100 kHz) linewidth in order to focus on other effects. In this subsection we vary the linewidth in conjunction with fiber length. As PAM4 performance is heavily determined by fiber length, this provides a good snapshot into regimes where PAM4 or DMT might have an advantage.

Figure 4 shows via color map, the ratio of PAM4 BER to DMT BER. We sweep fiber length from 5 to 50 km and laser linewidth from 10 kHZ to 2 MHz, a wide range of laser quality. Bandwidth is held at 25 GHz, and three bit rates are examined. These color
maps allow the selection of the best modulation for a given cost/quality of the laser source, and desired reach.

The region where DMT is best (red) is greatest at lowest bit rate, consistent with fixed linewidth results in Fig. 2. The region where PAM4 is best (blue) is largest at highest bit rate, again consistent with Fig. 2. Note that white regions reflect unachievable fiber lengths, that is, where the forward error correction (FEC) threshold is not respected by either PAM4 or DMT. Intermediate (yellow) areas in Fig. 4 shows the crossover between performance of DMT and PAM4. This crossover is slightly moving to higher fiber lengths as linewidth increases, because increasing laser linewidth degrades DMT performance.

V. CONCLUSION

We provide a tool to the systems designer with the choice of both wavelength bands (O vs. C band) and modulation formats. While DMT has inherently greater cost (DAC and FFT) than PAM, we have examined only IMDD DMT to level the playing field and constrain the additional complexity of DMT vis-à-vis PAM. We optimized the DMT signal by a judicious choice of the signal-to-carrier ratio. Extremely aggressive bit rates in bandwidth-constrained systems will tend to favor higher order PAM, but fiber reach will be very limited. When moderately aggressive rates (80 Gb/s in 25 GHz bandwidth) are acceptable, DMT is able to achieve much greater reach than PAM. While DMT reach is reduced when using lasers with MHz linewidths, the reach is still double that of PAM systems at any linewidth. The greatest reach attainable is always with DMT provided a low enough linewidth is available. Laser linewidth ultimately shifts the crossover point for performance of PAM vis-à-vis DMT.

REFERENCES


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