A design methodology for rock slopes susceptible to wedge failure using fracture system modelling

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ABSTRACT
This paper demonstrates how the use of fracture system modelling can be linked to limit equilibrium analysis of rock slopes susceptible to wedge failure. The use of fracture systems highlights some of the limitations inherent in traditional structural data analysis and representation. Consequently it allows for more comprehensive input data that can be used for stability analysis of rock slopes. In particular the developed methodology addresses important issues such as spatial variability and wedge size distributions. The paper introduces a series of guidelines for interpretation of the results of rock slopes. The proposed techniques arguably result in an improved level of confidence in the design of rock slopes susceptible to wedge failure.

KEYWORDS
Rock slopes; Wedge failure; Fracture system modelling; Limit equilibrium analysis; Discrete fracture networks

CITATION

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1 INTRODUCTION

The analysis and design of rock slopes susceptible to wedge failure is of interest to both civil and mining engineering. There are several tools of increasing complexity that can be used to facilitate a limit equilibrium analysis of rock slopes. Kinematic analyses are usually the first to be applied in order to establish whether a wedge can form under a specific structural configuration. Traditionally, the stability of a wedge is quantified by a factor of safety which is often defined as the ratio of forces tending to cause sliding to those that resist movement.

The importance of structure was recognised amongst others by Piteau (1972) when he suggested that “…stability of slopes in rock is determined principally by structural discontinuities in the mass and not by the strength of the rock itself”. It follows that a prerequisite for a rational analysis of rock slopes is the reliable application of a series of geological premises:

a) The characteristics of the fracture population (generic structural type, orientation and position in space, intensity, joint wall rock hardness and asperities, joint size, infilling and alteration materials between joint planes) can be assessed. As access to all fractures is not possible it is necessary to extract the maximum information. The description of the fracture population must be done on a sampling basis, and statistical analysis and judgment must be applied to decide whether the best estimate is made of the whole population.
b) Structural regions can be defined. This implies that the fracture population within the region has similar characteristics.

Coates (1981) was one of the earlier advocates of probabilistic techniques in rock slope stability as a way to better account for the inherent variability of input data. The probability of failure in a slope was defined as a function of both the probability of sliding and the probability of occurrence of a critical geometry. One of the earliest well documented studies on the probability of occurrence of wedges has been presented by Piteau and Martin (1977). Working at the Cassiar Mine open pit in British Columbia they employed probabilistic techniques to establish the probability of occurrence of unstable wedge failures spilling over the pit berms. This was accomplished by assuming normal distributions for the dip of each joint set. Einstein et al. (1983) provided the framework for linking persistence to geometry and spatial variability of fractures.

A major development in structural analysis of rock slopes was the development of block theory, Goodman and Shi (1985). Key block theory can be used to identify key blocks formed by different fractures. This implies that all blocks can either be infinite, without posing a threat to slope stability or finite blocks which can either be removable, or non removable because of their shape. Software codes such as SAFEX developed by Windsor and Thompson (1993), and KBSLOPE by Pantechnica (2001), can be used to undertake a limit equilibrium wedge analysis of key blocks. Um and Kulatilake (2001) provide a comprehensive case study for Shiplock slopes along the Three Gorges dam site in China. In this work they used block theory to identify key blocks and subsequently conducted a series of limit equilibrium stability analyses.

A major development in rock mass representation and visualisation has been the development of stochastic models for the representation of fracture systems. This has found numerous applications in rock engineering and will receive more attention given the wider availability of computation power. More recently Hadji georgiou and Grenon (2005), Rogers et al. (2006) have reported the results of slope stability analysis using a simulated rock mass. The use of fracture systems in combination with limit equilibrium techniques helps overcome limitations of traditional wedge analysis where fractures are assumed to be of infinite length, ubiquitous and are considered independent of each other.

It is realised that the quality of any slope stability investigation is influenced not only by the quality and quantity of field data but also by the way these data are used in any analysis. There is little immediate advantage in collecting comprehensive field data if these are subsequently ignored by the employed design tools. On the other hand the use of comprehensive models, requiring several input data, in the absence of adequate field data is also questionable. This paper illustrates this conundrum by means of a case study of a rock slope in Quebec City. Furthermore, it is recognised that the use of more sophisticated models based on quality field data is not in itself sufficient. There has to be a formalised method to quantify the applicability of a given model as well as to how best interpret the results. This paper introduces a series of processes that can be used to interpret the results of analysis. These vary from determining whether a wedge is potentially unstable to more rigid guidelines on the acceptability of failure.

2 DATA COLLECTION

On December 3, 1917, after almost 20 years of planning and construction, the Quebec Bridge opened for traffic. The Quebec Bridge has an important place in Canadian engineering as it collapsed during construction in 1907 killing 75 workers. A second attempt to construct the bridge in 1916 ended in thirteen workers losing their life after a partial collapse of the middle span of the bridge. Pearson and Delatte (2006) document the engineering and ethical lapses that led to the
collapse of the Quebec Bridge. The completed bridge is a major engineering feat and still stands as one of the world's great bridges. The Quebec Bridge is still a major attraction and serves commuters that live on the south shore of the St. Lawrence River. Reported rock falls along the road cut leading to the bridge have been the cause of some concern.

The exposed rock mass is part of the Cambrian sediments of the Chaudière Nappe, Fig. 1. The sedimentary succession within the nappe forms the Sillery group, which itself consists of three formations. The Saint-Nicolas formation consists of six lithofacies interpreted as recurring at various intervals in the stratigraphy of the formation. The formation can be divided into three units, a lower thickly bedded succession of greenish sandstone conglomerate with subordinate mudstone; a middle cyclic turbiditic sandstone–mudstone; and an upper predominantly red mudstone interval. The middle and upper units are tectonically repeated (from folding and thrust faulting) many times along the Chaudière River. As a whole the Saint-Nicolas Formation is a fining-upward unit which reflects a progressive deepening of the depositional setting from late rift-drift to the passive margin stages, Lavoie (2002). The exposed rock face is described as a thick pebbly green sandstone bed.

Fig. 1. Regional Geology of the Quebec City Bridge area, modified after Lavoie (2002).
Scanline mapping was undertaken along a 25.5 m long traverse oriented at a trend of 190° and a plunge of 00°. The slope had a height of 12 m and is oriented at a dip of 75° and a dip direction of 100°, Fig. 2. The rock mass is defined by polygonal blocks with planar fractures. This gives the appearance of a blocky rock mass as illustrated in Brown (1981). In situ block shape was not addressed further in this work, although it is the subject of recent work by Kalenchuk et al. (2006) where they suggest it can be used as a classification parameter for jointed rock masses. Analysis of the structural data identified four fracture sets, displayed in a stereographic projection, Fig. 3.

Fig. 2. The Quebec Bridge road cut.

Fig. 3. Contoured plot of structural data.
The characteristics of the discontinuity sets measured at the road cut are summarised in Table 1, where the mean dip, dip direction and 68% confidence interval cone are used to define the orientation of the identified fracture sets. The reported fracture frequency is along the scanline. Fracture size is arguably difficult to establish from field mapping data. It is recognised that the observed trace length is a function of the overall fracture plane. The mean trace length values reported in Table 1 were estimated based on the methodology proposed by Zhang and Einstein (1998). Scanline mapping has a built in bias. In this case study applying Terzaghi’s correction did not result any different fracture sets.

Table 1. The Quebec Bridge road cut structural field data

<table>
<thead>
<tr>
<th>Fracture Set</th>
<th>Dip/dip direction $(^\circ)$</th>
<th>68% confidence interval cone $(^\circ)$</th>
<th>Fracture frequency $(1/m)$</th>
<th>Mean trace length $(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>69/198</td>
<td>15</td>
<td>0.36</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td>65/007</td>
<td>15</td>
<td>0.48</td>
<td>5.5</td>
</tr>
<tr>
<td>3</td>
<td>54/114</td>
<td>13</td>
<td>0.68</td>
<td>7.5</td>
</tr>
<tr>
<td>4</td>
<td>89/246</td>
<td>13</td>
<td>0.20</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Fracture sets 1 and 2 are very widely spaced while fracture sets 3 and 4 are widely spaced. All 4 fracture sets display medium persistence, based on the ISRM classifications reported in Brown (1981). The exposed rock face is thus defined by medium persistence and a medium spacing fracture sets. The precision of the spacing estimates due to the relative small sample sizes was addressed following Priest (1993). Assuming that the spacings obey a negative exponential distribution, then the range within which the fracture frequency population mean at the 80% confidence level is: $0.25–0.66 \text{ m}^{-1}$ for fracture set 1, $0.35–0.78 \text{ m}^{-1}$ for fracture set 2, $0.16–1.00 \text{ m}^{-1}$ for 3 and $0.13–0.47 \text{ m}^{-1}$ for fracture set 4. The resulting precision is due to the size of the rock exposure.

3 STABILITY ANALYSIS

The Quebec Bridge road cut will be used to illustrate the influence of a specific analysis method on the stability of a rock slope susceptible to wedge failure. The following sections will review traditional methods of analysis based on deterministic approaches before addressing probabilistic and multiple wedge analysis based on fracture system generation. In order to illustrate the influence of structure and facilitate comparisons between the different methods it was decided not to vary the mechanical properties of fractures throughout the different types of analyses.

3.1 Kinematic and kinetic analyses

A kinematic analysis is normally the first stage in a rock slope investigation. Hudson and Harrison (1997) illustrate the necessary slope geometry and fracture orientation combination for a kinematically feasible wedge failure. The first condition is that the dip of the slope must exceed the dip of the line of intersection of the two fracture planes associated with the potentially unstable wedge. The second is that the line of intersection of the two fracture planes associated with the potentially unstable wedge must daylight on the slope plane.

A kinetic analysis indicates whether a kinematically feasible wedge will indeed move as a result of its low shearing resistance. The undertaken analysis for the exposed rock face identified
kinematically feasible wedges along the fracture set combinations: (1 and 2), (2 and 3) and (3 and 4), Fig. 4. A friction cone of 30° was used to define the frictional properties of all fracture sets.

Fig. 4. Stereographic representation of the necessary kinematic conditions for failure.

3.2 Deterministic analysis

As indicated in the previous section a kinematic analysis identifies the potential for wedge instability. An indication of the stability of a slope can be accomplished by the use of a deterministic limit equilibrium analysis that can provide a factor of safety. The mechanics of a sliding wedge in a rock slope have been described by Hoek and Bray (1981).

There are several wedge stability software available. In this work, the Swedge software (Rocscience, 2004) was used to determine the stability of individual wedges formed by the prevalent fracture sets identified in Table 1. Fig. 5 illustrates the largest possible wedge defined by geometrical constraints such as bench height (12 m) and maximum fracture size for the three fracture set combinations. The same mechanical properties were used for all fracture sets (a friction angle of 30° and cohesion of null). This analysis resulted in a safety factor of 0.42 for the wedge formed by fracture sets 1 and 3, a safety factor of 0.80 for fracture sets 2 and 3 and a safety factor of 0.42 for sets 3 and 4.

Although this type of analysis is used on a routine basis, it is somewhat limited in that it assumes that the resulting wedges are formed by ubiquitous fractures. Furthermore, it is not possible to evaluate the probability of occurrence of any of the wedges. In fact, the more sophisticated software packages such as Swedge try to address this limitation by allowing the user to “scale down” the resulting maximum wedge. This is accomplished by using a “recorded” trace length value and subsequently scaling down by the same factor all other sides of the wedge. This engineering approach, results in the definition of a more realistic wedge size that can be used in a deterministic analysis. Although this is arguably a better approximation, it still does not provide any indication of the probability density function (pdf) of the size of the defined wedge.
3.3 Probability of failure for a given wedge

The probability that any particular wedge is formed, and susceptible to failure, can be quantified by employing a Monte-Carlo analysis. The basis of such an approach is to use a statistical distribution to define one or more parameters that control the stability of a given wedge. The factor of safety is calculated based on “randomly” selected values from the statistical distributions that define the stability of a wedge. In this process it is possible to combine both deterministic and randomly selected input values. The process is repeated and results in a safety factor distribution for a given wedge.

The input data for the Quebec Bridge road cut have been presented in Table 1. In order to be able to compare the results obtained by the different approaches, fracture persistence and shear resistance were kept constant during the Monte-Carlo analysis. Fracture set orientation was allowed to vary according to a Fisher distribution. Based on 10,000 samplings, the probability of failure for the three wedge configurations is illustrated in Fig. 6 and summarised in Table 2. This table displays the probability of the actual factor of safety being less than 1.0, 1.3, 1.5, and 2.0. It also illustrates the mean factor of safety and the standard deviation for the 10,000 samplings.

Table 2. Probabilistic analysis for the rock slope

<table>
<thead>
<tr>
<th>Factor of safety</th>
<th>Wedge formed by fracture sets 1 and 3</th>
<th>Wedge formed by fracture sets 2 and 3</th>
<th>Wedge formed by fracture sets 3 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(F&lt;1.0))</td>
<td>0.94</td>
<td>0.65</td>
<td>0.86</td>
</tr>
<tr>
<td>(P(F&lt;1.3))</td>
<td>0.99</td>
<td>0.80</td>
<td>0.90</td>
</tr>
<tr>
<td>(P(F&lt;1.5))</td>
<td>1</td>
<td>0.89</td>
<td>0.93</td>
</tr>
<tr>
<td>(P(F&lt;2.0))</td>
<td>1</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>Mean</td>
<td>0.56</td>
<td>0.91</td>
<td>0.84</td>
</tr>
<tr>
<td>Standard</td>
<td>0.26</td>
<td>0.43</td>
<td>1.5</td>
</tr>
</tbody>
</table>
This implies that the wedge formed by the combination of fracture sets 1 and 3 is more likely to fail than the wedge formed by fracture set combination 2 and 3, or the wedge formed by fracture sets 3 and 4. This analysis demonstrated that the exposed rock face is susceptible to wedge failures, and is in agreement with field observations. This analysis, however, does not provide quantitative information on the probable size of the wedges that are formed or more importantly the probability of occurrence of these wedges. The high probabilities of failure and low safety factor for the Quebec Bridge road cut, Table 2 indicate instabilities. This is confirmed by the occurrence of rock falls.

4 FRACTURE SYSTEM MODELLING

Stochastic models provide powerful means of representing fracture systems based on available structural data. Fracture system modelling employs borehole, line mapping or face mapping field data to generate representative models of the prevailing structural conditions. Rogers et al. (2006) note that early interest in the fracture systems approach was associated with nuclear waste sites.
while in recent years there is increased focus towards the modelling of fractured hydrocarbon reservoirs with limited use in the design of rock engineering structures in fractured rock masses.

The fundamentals of stochastic modelling are discussed in detail by Dershowitz and Einstein (1988) where they demonstrate that fracture system models can be used to represent rock mass geometry as an entity. They furthermore present detailed descriptions of the Orthogonal, Baecher, Veneziano, Dershowitz and Mosaic Tessellation models. Staub et al. (2002) contributes to the discussion, by providing details on some other conceptual models that have been developed since and can be used for the modelling of fracture geometry. Table 3 lists the models reviewed by Dershowitz and Einstein (1988) and Staub et al. (2002).

Table 3. Main features of different fracture system models, modified from Staub et al. (2002)

<table>
<thead>
<tr>
<th>Model</th>
<th>Fracture characteristics considered in model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fracture Shape</td>
</tr>
<tr>
<td>Orthogonal</td>
<td>Rectangle</td>
</tr>
<tr>
<td></td>
<td>Unbounded</td>
</tr>
<tr>
<td></td>
<td>Unbounded</td>
</tr>
<tr>
<td>Baecher</td>
<td>Circle ellipse</td>
</tr>
<tr>
<td>Veneziano</td>
<td>Polygon</td>
</tr>
<tr>
<td>Dershowitz</td>
<td>Polygon</td>
</tr>
<tr>
<td>Mosaic Tessellation</td>
<td>Polygon</td>
</tr>
<tr>
<td>Enhanced Baecher</td>
<td>Polygon</td>
</tr>
<tr>
<td>Baecher algorithm, revised terminations</td>
<td>Polygon</td>
</tr>
<tr>
<td>Ivanova</td>
<td>Convex polygon</td>
</tr>
<tr>
<td>Poisson rectangle</td>
<td>Rectangle</td>
</tr>
<tr>
<td>Box Fractal model</td>
<td>Polygon</td>
</tr>
<tr>
<td>Geostatistical model</td>
<td>Polygon</td>
</tr>
<tr>
<td>War zone</td>
<td>Polygon</td>
</tr>
<tr>
<td>Non-planar zone</td>
<td>Polygon</td>
</tr>
<tr>
<td>Levy-Lee fractal</td>
<td>Polygon</td>
</tr>
<tr>
<td>Nearest neighbour</td>
<td>Polygon</td>
</tr>
<tr>
<td>Fractal POCS model</td>
<td>Polygon</td>
</tr>
</tbody>
</table>

The listed models assume that fractures are planar, and any location or autocorrelation process is possible. In most cases fracture locations are stochastic. Fracture size refers to trace length on two dimensional surfaces or as the surface area of individual fractures. Fracture sizes are usually stochastic either specified directly or indirectly through stochastic location and orientation. Bounding of fractures implies that fractures smaller than the region under consideration can be represented. Field observations suggest that fractures can either terminate at the intersection with other fractures or against intact rock. This is recognised in most models. Co-planarity implies that a number of fractures can be located in the same plane.

The majority of the models have not been adequately verified for several engineering applications. In practice, the choice of the model will depend on how it can be related to the available field data and to the engineering needs of the project. Recent years have seen the development of several fracture systems generators of varying complexity and ease of use. These generators can be model specific, such as Stereoblock which was based on the Baecher model, Hadjigeorgiou et al. (1995), or the one used for this analysis based on the Veneziano model. On the other hand, FracMan is a commercially available software that has integrated a broad selection of fracture system models that can capture different geological environments and can be used for diverse engineering applications, Dershowitz (2007).
4.1 Fracture system generation

Previous work by the authors relied on fracture generators based on the Baecher model, Grenon and Hadjigeorgiou (2003). More recently and in the analysis of rock slopes the authors have chosen to use a fracture system generator based on the Veneziano model, Hadjigeorgiou and Grenon (2005). The Veneziano model is described in Dershowitz and Einstein (1988). The model relies on the generation of a Poisson network of planes in 3D space followed by a secondary process of tessellation by a Poisson line process and marking of polygonal fractures. The resulting polygonal shape fractures, and the implication that fractures produced on the same plane during the primary generation process remain coplanar after the secondary tessellation process are often perceived as the main limitations of the Veneziano model. Nevertheless, the Veneziano model is conceptually simple and the required input parameters can be easily inferred from field data. In this work, modifications were made to the Veneziano model to consider non coplanar fractures. It is possible to introduce both fracture set hierarchy and zoning but in this case study this was not required by the field data. Furthermore, the developed, modified Veneziano model can consider discretely located fractures.

There are other conceptual models that aim to interpret not only geometry but also fracture hierarchy. Examples of these more sophisticated models can be found in the work of Ivanova (1998). In the present approach the aim was to develop a reliable geometric model that would facilitate more complex rock slope stability analyses. This is justified given the size of fractures with respect to the excavation size. The information in Table 1 was used to generate several three dimensional fracture systems in a $50 \times 50 \times 50$ m volume, Fig. 7a. All generated fractures are of a polygonal shape. The generation is realised on a set basis, using as input fracture size, intensity, orientation and co-planarity.

Fig. 7. (a) Fractures of the final simulated system (50 m by 50 m by 50 m). (b) A section cut of the simulated system parallel to the slope face (50 m by 50 m).
A basic question is the reliability of the generated models. In other words, are the generated models really representative of field conditions? This is difficult to verify in that, even at the best conditions there are only limited field data. To overcome this inherent limitation the authors have relied on a simple approach, whereby the generated fracture system is subsequently sampled. Consequently, the results of the sampled virtual rock mass are compared to the field data.

The generated system was calibrated using forward modelling. A section plan of the simulated fracture system, parallel to the field exposure, is provided in Fig. 7b. The fractures on the section plan were sampled using a scanline defined by the same orientation as the one in the field. The resulting fracture sets of the simulated rock mass are illustrated in Fig. 8 and the structural properties of the generated fracture sets are summarised in Table 4. This has facilitated a comparison with the original field data and validates the generated fracture systems.

Comparing the results from Table 4 to those of the field data in Table 1, as well as the corresponding stereonets, corroborates the validity of the simulated data for a given system. This process was repeated for all the generated systems. The variations in fracture frequency for any set of simulations is typically less than 4% and for mean trace length less than 10%. It follows that all 150 simulations are plausible representations of the in situ fracture system.

Fig. 8. The major sets of the Quebec Bridge road cut (modelled data).

<table>
<thead>
<tr>
<th>Fracture set</th>
<th>Dip/dip direction $^\circ$/$^\circ$</th>
<th>68% confidence interval cone $^\circ$</th>
<th>Fracture frequency $1/m$</th>
<th>Mean trace length $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71/194</td>
<td>19</td>
<td>0.33</td>
<td>5.1</td>
</tr>
<tr>
<td>2</td>
<td>64/006</td>
<td>19</td>
<td>0.40</td>
<td>5.5</td>
</tr>
<tr>
<td>3</td>
<td>50/114</td>
<td>16</td>
<td>0.70</td>
<td>8.2</td>
</tr>
<tr>
<td>4</td>
<td>83/249</td>
<td>17</td>
<td>0.18</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Table 4. The Québec Bridge road cut modelled data
4.2 Wedge stability

Having generated a fracture system it is now possible to introduce any actual slope configuration. This allows one to identify all kinematically possible wedges, Fig. 9. The definition of the location, size, etc. for every individual wedge is then possible. The same type of limit equilibrium stability analysis that was employed in the deterministic analysis is used to determine the stability of every wedge formed across the crest of the slope.

The developed model is flexible enough to accommodate the inherent variability of material properties. These can be best tackled by selecting appropriate pdf. In this study however, the aim was to investigate the impact of fracture orientation and its spatial variability. In order to facilitate this comparison the same material strength parameters were employed for the deterministic, probabilistic and stochastic approaches. This involved keeping the material properties as cohesionless and the angle of friction equal to 30°.

The following sections intend to illustrate that the generation of a more realistic fracture system is justified by the further insight gained on the stability of a rock slope by further analysis and interpretation of the results.

![Fig. 9. Tetrahedral wedges formed along the crest in a simulation.](image)

4.2.1 Fracture set combinations that result in wedges along the crest of a slope

This section summarises the results of a series of wedge stability analyses along the crest of a slope in a generated fracture system. These wedges are illustrated in Fig. 10, where the same fracture set combinations (1, 3) (2, 3) and (3, 4) are critical as already identified by the deterministic
analyses and reported in Section 3.3. Fig. 10, however, provides further useful information. 58 wedges were formed along the crest of the slope with 23 being stable and the remaining 35 unstable. A particular wedge is considered as stable if the resulting factor of safety is greater than 1. It is now possible to recognise that the majority of the wedges are created by the combination of fracture sets 2 and 3. The influence of fracture set combinations 1 and 3, and 3 and 4, is considerably smaller. In Fig. 10 there is a stable wedge configuration defined by fracture set (3, 3). These are wedges formed by the same fracture set (3) as a result of variations in dip and dip direction. These wedges tend to be thin and elongated and are not a source of concern.

Of further importance is the observation that all wedges formed by fracture sets 3 and 4 are in fact unstable with P(F)= 100%, while the combination of fracture sets 2 and 3 results in 54% of wedges along the crest of the slope being unstable. Finally, 78% of the wedges attributed to fracture sets 1 and 3 are unstable. These results are in agreement with the Monte-Carlo analysis presented in Section 3.3. It should be noted that the fracture system analysis has identified the previously not recognised possibility of unstable wedges formed by the interaction of fracture sets 1 and 4.

The number of wedges formed by different fracture set combinations is not in itself an adequate measure of the stability of a given rock exposure. The size and weight of each wedge are arguably also important. This is demonstrated in Fig. 11 where the scatter plot presents the relation between fracture set combinations and resulting weight of the resulting wedges. This particular bit of information could not have been readily obtained by traditional deterministic or probabilistic analyses such as those described in Section 3. In reference to Fig. 11 it is noted that the larger wedges result from the combination of fracture sets 2 and 3.
5 RELIABILITY ANALYSIS

The undertaken stochastic fracture system approach has made it possible to conduct a more comprehensive analysis and treatment of the available structural data than previously possible using traditional techniques. A realistic rock mass model, such as developed from fracture system modelling, allows one to better understand the influence of variations in the input data and arguably facilitates the interpretation of the results of slope stability investigations.

It is recognised that there are inherent variations between each simulation of a stochastic system. This poses the question on the minimum number of simulations to ensure that there is good representation of the range of possible situations or conditions in the system. Once this is established it is possible to establish the degree of confidence that we may assign to a stability analysis.

5.1 Selecting the number of simulations and necessary design criteria

Integrating the fracture system generation to wedge stability has allowed the determination of the pdf of a wedge size. It follows that each simulation and analysis, due to the stochastic nature of fracture system modelling, will result in variations in the resulting wedge size. In order for the proposed methodology to gain wider acceptability for design purposes it is necessary to establish what constitutes an acceptable number of simulations that will result in consistent results. This is addressed in Fig. 12 where a series of curves linking wedge size to the number of simulations are plotted. These curves, however, are site specific and in this case study it was assumed that the wedges were cohesion less. The wedge weight of unstable wedges can then be associated with any series of design criteria that may be pertinent for a particular slope. In this case study it was decided to consider the following thresholds \(90.0\%\), \(95.0\%\), \(99.0\%\), and \(99.5\%\) probability for generating unstable wedges smaller than a given size.

![Fig. 12. Unstable wedge size versus number of simulations for different acceptance criteria.](image)

A visual assessment of Fig. 12 suggests that at least 100 simulations are necessary before accepting the results for \(90.0\%\), \(95.0\%\), and \(99.0\%\) and \(99.5\%\) thresholds. This approach arguably provides a crude visual criterion that can be used to accept and validate these simulations. It follows
that there is an interest and need to develop a more rigorous approach for establishing the number of simulations that result in converging results.

Defining “\(\nu\)” as the value obtained for a given simulation “\(n\)” and “\(\nu_{\text{mean}}\)” as the mean value for the immediate preceding 20 simulations then it is possible to set an acceptable level of variation for these two values. In this case study, it was decided that \(\frac{\nu}{\nu_{\text{mean}}}\) should be between 0.995 and 1.005. If this was the case, for five consecutive simulations then the number of simulations was deemed sufficient to provide for consistent results. Referring to Fig. 12, acceptable results of the maximum wedge weight were obtained after 56 simulations with 90.0% of the simulations complying. It only took 57 simulations for 95.0%, and 130 simulations for 99.0% compliance. Finally, 99.5% compliance was achieved after 134 simulations. It follows that in this case study 134 simulations would satisfy all criteria.

5.2 Probabilistic analysis and interpretation based on multiple simulations

Following up on the reliability analysis, outlined in the previous section, 150 simulations were undertaken. The slope configuration was introduced in these simulated rock masses and the wedges formed along the crest of the slope were identified. The resulting probability of failure for the different fracture set combinations for the 150 simulations is summarised in Table 5 with reference to the probability of the actual factor of safety being less than 1.0, 1.3, 1.5, and 2.0. The mean factor of safety and the standard deviation for the defined wedges was also determined. The fracture system approach resulted in relative probability of failure of 87% for sets 1 and 3, 57% for sets 2 and 3, and 90% for fracture sets 3 and 4. The wedges formed by the combination of fracture sets 2 and 3 results in lower probability of failure and a higher mean factor of safety than wedges formed by the other fracture set combinations.

<table>
<thead>
<tr>
<th>Factor of safety</th>
<th>Wedge formed by fracture sets 1 and 3</th>
<th>Wedge formed by fracture sets 2 and 3</th>
<th>Wedge formed by fracture sets 3 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(F&lt;1.0))</td>
<td>0.87</td>
<td>0.57</td>
<td>0.90</td>
</tr>
<tr>
<td>(P(F&lt;1.3))</td>
<td>0.93</td>
<td>0.78</td>
<td>0.92</td>
</tr>
<tr>
<td>(P(F&lt;1.5))</td>
<td>0.97</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>(P(F&lt;2.0))</td>
<td>0.99</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>Mean</td>
<td>0.66</td>
<td>1.0</td>
<td>0.68</td>
</tr>
<tr>
<td>Standard</td>
<td>0.35</td>
<td>0.54</td>
<td>0.62</td>
</tr>
</tbody>
</table>

The interest in the fracture system approach is that it can provide much more information than traditional probabilistic approaches. In Fig. 13 it can be seen that the frequency of unstable wedges formed by fracture sets 2 and 3 is considerably higher than the other sets. In other words, this is a much more critical combination in that the number of unstable wedges formed is greater. This information could not have been captured from traditional probabilistic analyses. In this case study the number of wedges defined by fracture sets 2 and 3 was critical. This was not identified in the preceding probability of failure analysis outlined in Section 3.3.

Fig. 14 plots weight of formed wedges versus fracture pair for 9436 wedges created during the 150 generated systems. The vast majority of bigger wedges are formed by the combination of fracture sets 2 and 3. The interaction of fracture sets (3, 3) and (1, 3) also contributes to the formation of large wedges.

Although probabilistic slope stability analysis can be used to identify the accepted level of design it can not, however, be used for establishing an absolute design (certainty) value. The choice of
which acceptable confidence intervals is used depends on the importance of the slope and the potential consequences of failure. The developed approach is capable to quantify any selected geotechnical criteria as required. In practice the selected criteria will be dictated by the use, and importance of the roadway. This is intuitively the approach in road design where parameters such as average vehicle risk, road width, climate etc. have to be taken into consideration. It was illustrated in Fig. 12 that several runs are necessary to establish the representative wedge weight for different design thresholds (90, 95, 99, and 99.5%). From an engineering perspective, the designer can either accept the consequences of any of the developed probability guidelines or modify the design to better respond to these standards. This allows for a more flexible design strategy. This approach was further developed in order to explore the relationship between wedge frequency and weight along the slope crest. In Fig. 15 the number of wedges formed along 100 m of crest, taking into consideration the maximum weight of the wedges formed was plotted against the number of undertaken simulations. A visual assessment of Fig. 15 suggests that consistent results are arrived at after 76 simulations. This can be further quantified by applying the same convergence criteria as outlined in Section 5.1. It follows that in this case study 145 simulations would satisfy all criteria including the number that would account for the unlikely event of a 100 ton wedge. It is interesting to note that if a small number of simulations were undertaken then the interpretation of the results would have been limited.

Fig. 13. Influence of fracture sets in defining stable and unstable wedges for 150 systems.

Fig. 14. Fracture set combinations versus wedge weight (stable and unstable wedges) for 150 systems.
Once the design has been deemed acceptable from a numerical point of view it is then possible to explore the new information. For example it is clear in Fig. 15 that there are seven, 5 tonnes wedges likely to be present along a 100 m crest, while there are possibly twenty, 1 ton wedges along the same length. This information is particularly useful in the design of reinforcement and containment strategies for a particular slope.

The developed methodology has identified that it is possible to obtain much more information than previously available with deterministic or traditional probabilistic techniques. It is now possible to determine and quantify wedge size distributions, and frequency. This in itself has been the motivation in developing further stability recommendations for the design of slopes susceptible to wedge failure. This was presented in the preceding section where also the numbers of simulations to arrive at consistent results were discussed.

6 CONCLUSIONS

Discrete wedge failure along rock slopes is a common phenomenon. This paper traces the evolution of tools used for limit equilibrium analysis and design of rock slopes susceptible to wedge type failure. The driving force has been the need to capture and represent the structural information and to arrive at more realistic prediction on the stability of natural and man made excavations.

In the investigated road exposure of the Quebec Bridge it was noted that kinematically feasible wedges were formed along the following fracture set combinations: (1 and 2), (2 and 3) and (3 and 4). A deterministic analysis suggested that the resulting wedges were unstable (FS < 1). This resulted in a safety factor of 0.42 for the wedge formed by fracture sets 1 and 3, a safety factor of 0.80 for fracture sets 2 and 3 and a safety factor of 0.42 for sets 3 and 4. A probabilistic analysis whereby the fracture set orientation was allowed to vary according to a Fisher distribution was performed. Based on 10,000 samplings, the probability of failure for the three wedge configurations varied for the different fracture set combinations with the wedge formed by the combination of fracture sets 1 and 3 being more likely to fail than the wedge formed by fracture set combination 2 and 3, or the wedge formed by fracture sets 3 and 4. This information could not have be determined by a deterministic limit equilibrium analysis.

The generation of a fracture system for this excavation demonstrated that fracture sets (2 and 3) had a higher probability of occurrence and were larger in size. In this respect they were a major concern on the stability of the slope. The inherent variation in the results of every simulation was addressed developing a criterion to ensure that a fracture system-limit equilibrium stability analysis provides a good representation of the range of possible situations or conditions in the system.
This paper has used a case study to illustrate each approach and in particular to elaborate the use of fracture systems. Fracture systems provide a more holistic approach to the stability of a slope than of a particular wedge. On the other hand if fracture systems are to be used to their full potential it is recognised that there is a need for further development of interpretation tools. This paper provides preliminary guidelines in the interpretation of wedge stability results using fracture systems.

The choice of acceptable confidence intervals will arguably depend on the importance of the slope and the potential consequences of failure. The developed approach is capable to quantify any selected geotechnical criteria. In practice these will be dictated by the use, and importance of the roadway. This is intuitively the approach in road design where parameters such as average vehicle risk, road width, climate etc. all have to be taken into consideration.

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