Slope orientation assessment for open-pit mines, using GIS-based algorithms

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ABSTRACT
Standard stability analysis in geomechanical rock slope engineering for open-pit mines relies on a simplified representation of slope geometry, which does not take full advantage of available topographical data in the early design stages of a mining project; consequently, this may lead to nonoptimal slope design. The primary objective of this paper is to present a methodology that allows for the rigorous determination of interramp and bench face slope orientations on a digital elevation model (DEM) of a designed open pit. Common GIS slope algorithms were tested to assess slope orientations on the DEM of the Meadowbank mining project’s Portage pit. Planar regression algorithms based on principal component analysis provided the best results at both the interramp and the bench face levels. The optimal sampling window for interramp was 21 x 21 cells, while a 9 x 9-cell window was best at the bench level. Subsequent slope stability analysis relying on those assessed slope orientations would provide a more realistic geometry for potential slope instabilities in the design pit. The presented methodology is flexible, and can be adapted depending on a given mine’s block sizes and pit geometry.

KEYWORDS
Open-pit mining, Interramp orientation, Bench face orientation, GIS, Integrated design, Block modeling

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1 INTRODUCTION

The creation of rock slopes in open-pit mines results from the inputs of planning, production, and geomechanics groups. According to Hustrulid (2000), both good planning and good geomechanics are necessary for the preparation of good designs. Good production is required to ensure that the “as-built” slopes closely resemble the “as designed” slopes.

The planning group now routinely uses software tools for assessing geology, mineral resources, ultimate pit, and mine planning to provide plans and layouts to the production group. The geomechanics group’s contribution is not fully integrated in the workflow of rock slope creations because of the inability to perform stability analysis with compatible software tools.

Some efforts have been made to integrate geomechanical design into commercially available mine design tools, such as the Stereonet Viewer and Terrain modules in Datamine (Datamine, 2010), and the Geotechnical Tools module in Vulcan (Maptek, 2010). However, while useful, these modules are not commonly used and they cannot perform complex stability analysis. More recently, Grenon and Hadjigeorgiou (2010) integrated a probabilistic limit equilibrium approach into a
commercially available design tool, Gemcom Surpac (Gemcom, 2010). Although enabling more
detailed deterministic and probabilistic limit equilibrium analysis within a mine planning software tool,
this approach does not take full advantage of the three-dimensional representation of the planned
pit geometry for assessing slope orientations.

A more complete representation of pit geometry should comprise the three main components of
an open-pit slope design: overall slope angle, interramp angle, and bench face angle (Fig. 1). The
overall pit slope angle is from crest to toe and incorporates all ramps and benches. The interramp
angle of the slope is defined as the slope lying between each ramp. The face angle of individual
benches depends on the bench height, or combined multiple benches, and the width of benches
required to contain minor rock falls (Wyllie and Mah, 2004). These angles may vary around the pit
to accommodate geology and/or planning considerations.

Fig. 1. Open-pit slopes after Wyllie and Mah (2004).
In hard rock open-pit mines, the most common slope stability issues at the bench and interramp levels are structurally controlled. Typical stability analysis involves kinematic and limit equilibrium analysis for planar, wedge, and toppling failure modes (Fig. 2). The slope geometry is usually considered planar and constant over the studied area.

Currently, geologists and mining engineers use block modeling at the prefeasibility, feasibility, and full mine production stages in assessing mineral resources, mining reserves, and final pit layouts. Fig. 3 presents the topography of the final pit of a mining project. Pit topography is defined by blocks, as in Fig. 3a. The very strict rules governing reserves and resources estimation in the mining business ensure that block sizes are small enough to very accurately define the pit topography. A plan view representation could also be used in defining the pit, as in Fig. 3b. The cells defining the ultimate pit surface can be defined by their center x, y, and z coordinates. This cellular representation of the pit topography is equivalent to the Digital Elevation Model (DEM), a Geographical Information System (GIS) raster layer representing elevation. Usually, cell size in GIS analyses is limited by the data acquisition method (orthophoto, laser scanning, etc.), and/or by the raster layer’s intended usage. In mine block modeling, no photos or laser surveys of the final pit are available a priori. Cells or block sizes are dictated by the diamond drilling holes (DDH) pattern used

Fig. 2. Typical failure modes: (a) planar, (b) wedge and (c) toppling, modified from Hudson and Harrison (1997).
in defining the mineralization within the orebody. All subsequent analyses are limited by the cell or block sizes dictated by the ore resources estimation process.

Fig. 3. (a) Block modeling representation of a 3D pit, (b) Cellular representation of the pit surface in 2D.

At prefeasibility and feasibility stages, the best practice lays in obtaining structural information from outcrop, drill cores and oriented drill cores mapping. This information is used to build a structural model of the pit area. At these design stages, the target levels of data confidence for structural models are 40–50 and 45–70% for prefeasability and feasibility, respectively (Read and Stacey, 2009). This structural information can be stored easily within a block model generated with a mine design tool (Read and Stacey, 2009; Grenon and Hadjigeorgiou, 2010).

Adequate slope orientation determination is necessary to better integrate the work of the geomechanical group into the slope creation process. This paper will present a formal methodology to compute slope orientation at the interramp and bench levels within mine design software tools relying on block modeling. The applicability of the most commonly available GIS algorithms for determining slope orientation will be reviewed from a mining engineering perspective. The Meadowbanks open-pit case study will be used for validating the applicability of the various algorithms and for evaluating the most appropriate methods. The slope orientations thus obtained would arguably be the best suited for assessing slope stability in subsequent stability analyses of pit slopes.

2 SLOPE ORIENTATION

This section presents the most common slope algorithms used to compute slope orientation as a local property of the DEM. Section 3 shows how the slope algorithms are used to compute slope orientation on the DEM of an existing open-pit mine.

2.1 Slope orientation terminology

Slope orientation may be defined in various ways. The vocabulary used by geologists, mining engineers, geotechnical engineers, and GIS specialists can differ significantly. This paper uses mining rock mechanics engineers’ terminology (Fig. 4).
Most commonly, the orientation of a plane may be defined by two angles:

1. **Dip**: the maximum inclination of a plane to the horizontal.
2. **Dip direction**: the direction of the horizontal trace of the line of dip, measured clockwise from north.

Alternatively, two angles defining a line can be used:

1. **Plunge**: the dip of the line with a positive plunge below the horizontal.
2. **Trend**: the direction of the horizontal projection of the line measured clockwise from North.

### 2.2 Algorithms

Skidmore (1989) and Jones (1998), among others, reviewed and compared slope algorithms for natural mountain slopes. These algorithms use the elevation values of the neighboring cells to compute slope orientation parameters. These algorithms mostly use a $3 \times 3$-cell moving sampling window centered on the cell where slope orientation is computed. This section presents these algorithms.

The method of Unwin (1981) is a second-order, finite-difference algorithm that computes dip and dip direction from the nearest four elevation points on the DEM. Considering a sampling window around cell $z_8$

\[
\begin{pmatrix}
  z_7 & z_1 & z_3 \\
  z_8 & z_3 & z_5
\end{pmatrix}
\]

The first partial derivative of cell $z_8$ with respect to $x$ is given by

\[
\frac{dz}{dx} = \frac{(z_3 - z_7)}{2\Delta x}
\]

where $\Delta x$ is the cell dimension along the $x$ axis.

The first partial derivative of cell $z_8$ with respect to $y$ is given by

\[
\frac{dz}{dy} = \frac{(z_5 - z_1)}{2\Delta y}
\]
where $\Delta y$ is the cell dimension along the y axis.

Dip and dip direction are then calculated according to the following relationships:

\[
dip = a \tan\left(\sqrt{(dz/dx)^2 + (dz/dy)^2}\right)
\]
\[
dip \text{ direction} = a \tan\left(\frac{(dz/dx)}{(dz/dy)}\right)
\]

where $dz/dx$ is the first partial derivative of elevation with respect to the x axis, and $dz/dy$ is the first partial derivative of elevation with respect to the y axis.

The method of Sharpnack and Akin (1969) is a third-order, finite-difference algorithm that computes dip and dip direction from the nearest eight elevation points on the DEM. Considering a sampling window around cell $z_8$ of:

\[
\begin{bmatrix}
z_0 & z_1 & z_2 \\
z_7 & z_8 & z_3 \\
z_6 & z_5 & z_4
\end{bmatrix}
\]

The first partial derivative of cell $z_8$ with respect to x is given by

\[
(dz/dx) = \left(\frac{(z_2 + z_3 + z_4) - (z_0 + z_6 + z_7)}{6\Delta x}\right)
\]

where $\Delta x$ is the cell dimension along the x axis.

The first partial derivative of cell $z_8$ with respect to y is given by

\[
(dz/dy) = \left(\frac{(z_4 + z_5 + z_6) - (z_0 + z_1 + z_2)}{6\Delta y}\right)
\]

where $\Delta y$ is the cell dimension along the y axis.

Dip and dip direction are then calculated according to Eqs. (3) and (4). The approach proposed by Sharpnack and Akin (1969) is similar to a planar regression on these eight elevation points.

The methodology proposed by Horn (1982) involves applying weighting coefficients to some of the terms of the Sharpnack and Akin (1969) algorithm. The weightings are inversely proportional to the distance from the center cell. Consider a sampling window around cell $z_8$ of:

\[
\begin{bmatrix}
z_0 & z_1 & z_2 \\
z_7 & z_8 & z_3 \\
z_6 & z_5 & z_4
\end{bmatrix}
\]

The first partial derivative of cell $z_8$ with respect to x is given by

\[
(dz/dx) = \left(\frac{(z_2 + 2z_3 + z_4) - (z_0 + 2z_6 + z_7)}{8\Delta x}\right)
\]

where $\Delta x$ is the cell dimension along the x axis.

The first partial derivative of cell $z_8$ with respect to y is given by

\[
(dz/dy) = \left(\frac{(z_4 + 2z_5 + z_6) - (z_0 + 2z_1 + z_2)}{8\Delta y}\right)
\]

where $\Delta y$ is the cell dimension along the y axis.

Dip and dip direction are then calculated according to Eqs. (3) and (4). This algorithm is implemented in the widely used and commercially available GIS software tool ArcGIS (ESRI, 2010).
As stated previously, the method proposed by Sharpnack and Akin (1969) is identical to a planar regression fitted to eight-grid cells. Travis et al. (1975) proposed approaches using planar regressions fitted to nine-grid cells. Fernandez (2005), although not applying it to a DEM, suggested the use of planar regression and moment of inertia analyses for computing plane dip and dip direction of a set of points belonging to the same plane. In Fernandez (2005) the number of points is not limited to 9.

In this paper, principal component analysis (PCA) was used to fit a planar regression that minimizes the perpendicular distances from the elevation data to the fitted plane. With principal component analysis, a large number of independent variables can be systematically reduced to a smaller, conceptually more coherent set of variables (Dunteman, 1989).

In this approach, the sampling window is not limited to nine cells but can be of any size. The sampling window is centered on the cell where the slope orientation is computed. In this paper, we use the PCA methodology, presented in the Matlab statistics toolbox (MathWorks, 2010), to fit a plane on a set of 3D points.

The following equation was solved:

\[
\begin{pmatrix}
\text{var}(x) & \text{cov}(x,y) & \text{cov}(x,z) \\
\text{cov}(y,x) & \text{var}(y) & \text{cov}(y,z) \\
\text{cov}(z,x) & \text{cov}(z,y) & \text{var}(z)
\end{pmatrix}
\begin{pmatrix}
v_{1x} \\
v_{1y} \\
v_{1z}
\end{pmatrix}
\begin{pmatrix}
v_{2x} \\
v_{2y} \\
v_{2z}
\end{pmatrix}
\begin{pmatrix}
v_{3x} \\
v_{3y} \\
v_{3z}
\end{pmatrix}

= \begin{pmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{pmatrix}
\begin{pmatrix}
v_{1x} \\
v_{1y} \\
v_{1z}
\end{pmatrix}
\begin{pmatrix}
v_{2x} \\
v_{2y} \\
v_{2z}
\end{pmatrix}
\begin{pmatrix}
v_{3y} \\
v_{3x} \\
v_{3z}
\end{pmatrix}
\]

\[\text{(9)}\]

where \(x, y, \) and \(z\) are the Cartesian coordinates of the cells considered, \(\nu\) is the matrix of the column vectors representing the coefficients of the principal components, and \(\lambda_i\) are the variances of the principal components, such as \(\lambda_1 > \lambda_2 > \lambda_3\).

The first two columns of the PCA matrix \((\nu)\) define vectors forming a basis for the plane. The third column vector of the matrix defines a vector orthogonal to the first two, and its coefficients define the normal vector of the plane. Finally, dip and dip direction were derived according to the following relationships:

\[
\text{dip} = 90 - \left( a \tan \left( \frac{v_{3z}}{\sqrt{(v_{3x})^2 + (v_{3y})^2}} \right) \right)
\]

\[\text{(10)}\]

\[
\text{dip direction} = a \tan \left( \frac{v_{3x}}{v_{3y}} \right)
\]

\[\text{(11)}\]

3 CASE STUDY

The various algorithms presented in Section 2 will be tested on the DEM of the Portage pit. The Portage pit is part of the Meadowbank mining project. It is located in the Canadian Far North district of Kivalliq, in the Nunavut territory, about 70 km north of Baker Lake village. The Meadowbank mining project comprises four gold ore bodies. According to Agnico-Eagle Mines Limited, as of 2009, three of them were planned to be mined: The Portage, Goose Island, and Vault orebodies. The Portage pit started production early in 2010, and it has probable reserves of 3.6 million ounces of gold.

The Portage deposit is in highly deformed, magnetite rich iron-formation rocks. The Portage gold deposit is defined over a 1.85 km strike length and cross-lateral extensions ranging from 100 to 230 m. The geometry of the orebody consists of a NNW striking recumbent fold with limbs that extend to the west. The mineralization in the lower limb of the fold is typically 6–8 m in true thickness, reaching up to 20 m in the hinge area of the fold.

Pit optimization was performed to define the ultimate pit for the Portage project. The ultimate pit is simply the pit limits presenting the greatest net present value (NPV). This type of optimization is performed in standard mine design software tools. Besides economic parameters, such as mining
and milling costs and selling price, geometrical parameters must be defined as input parameters in any pit optimization analysis. During the initial optimization of the Portage pit, different slope angles were used for different regions of the pit based on the results of preliminary slope stability analyses. These angles were used as upper bound angles during the optimization process. Fig. 5 presents the various slope angles used during this process. For most of the pit, the limiting upper bound interramp angle is either 50° or 55° while the bench face angle is either 70° or 80°.

Fig. 3a presents a 3D view of the ultimate pit defined within the block model. Fig. 3b shows a plan view of the ultimate pit. A total of 106,354 cells define the ultimate pit topography in plan view. All cells are defined by the x, y, and z coordinates of their centers.

![Fig. 5. Slope angles used during pit optimization.](image)

3.1 Slope orientation assessment

Using the final pit DEM defined by 106,354 cells, slope orientations at the interramp and bench face levels were assessed. The block size defining the block model is 2.5 m x 2.5 m x 2.5 m, and thus the cells were 2.5 m x 2.5 m in size.

3.2 Interramp level

Slope orientation was first assessed at the interramp level. Four methods were used to assess these orientations on the entire pit. The Unwin (1981), Horn (1982), and Sharpnack and Akin (1969) algorithms were used. Furthermore, the PCA planar regression approach with various window sizes was used to calculate slope orientations. All dip and dip direction results were rounded to the nearest integer. Slope orientation values were calculated at each of the 106,354 cells. The results from the four approaches were compared using various techniques.

Table 1 presents the number of unique slope orientations defining the pit topography obtained from the various algorithms. These unique orientations are tabulated as the number of unique dip values, the number of unique dip direction values, and the number of unique orientation values (combining dip and dip direction) that define the pit topography for the various algorithms. The smaller sampling window of the Unwin (1981), Horn (1982), and Sharpnack and Akin (1969) algorithms defines a very small number of unique slope orientation combinations. This is explained by the uniform size of the DEM cells (2.5 m x 2.5 m) and by the uniform block size used in mine design (2.5 m x 2.5 m x 2.5 m). Thus, when using a small number of cells to compute slope orientation, a very limited range of possible slope orientation outcomes can be computed. This is not consistent with the pseudo-oval shape of an open pit that has a 360° variation in the dip direction.
of slope walls. Larger sampling windows reflect a greater variability in slope orientation. This is because larger sampling windows will intercept a greater number of cells. The computed slope orientations will reflect the number of cells considered in the analysis. The larger the sampling window, the less probable it is to obtain two identical windows (defined by similar Cartesian coordinates). Thus, the number of unique slope orientation combinations will be greater when using larger windows.

### Table 1. Slope orientation determination

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of cells</th>
<th>Number of unique dip values</th>
<th>Number of unique dip direction values</th>
<th>Number of unique orientations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unwin (1981)</td>
<td>106,354</td>
<td>27</td>
<td>172</td>
<td>299</td>
</tr>
<tr>
<td>Sharpnack and Akin (1969)</td>
<td>106,354</td>
<td>50</td>
<td>335</td>
<td>1555</td>
</tr>
<tr>
<td>Horn (1982)</td>
<td>106,354</td>
<td>51</td>
<td>336</td>
<td>1513</td>
</tr>
<tr>
<td>PCA 3 x 3</td>
<td>106,354</td>
<td>70</td>
<td>359</td>
<td>5736</td>
</tr>
<tr>
<td>PCA 5 x 5</td>
<td>106,354</td>
<td>81</td>
<td>361</td>
<td>16,844</td>
</tr>
<tr>
<td>PCA 7 x 7</td>
<td>106,354</td>
<td>78</td>
<td>361</td>
<td>19,608</td>
</tr>
<tr>
<td>PCA 9 x 9</td>
<td>106,354</td>
<td>74</td>
<td>361</td>
<td>19,529</td>
</tr>
<tr>
<td>PCA 13 x 13</td>
<td>106,354</td>
<td>69</td>
<td>361</td>
<td>18,717</td>
</tr>
<tr>
<td>PCA 17 x 17</td>
<td>106,354</td>
<td>63</td>
<td>361</td>
<td>18,259</td>
</tr>
<tr>
<td>PCA 21 x 21</td>
<td>106,354</td>
<td>62</td>
<td>361</td>
<td>17,849</td>
</tr>
<tr>
<td>PCA 25 x 25</td>
<td>106,354</td>
<td>61</td>
<td>361</td>
<td>17,316</td>
</tr>
<tr>
<td>PCA 29 x 29</td>
<td>106,354</td>
<td>60</td>
<td>361</td>
<td>16,941</td>
</tr>
<tr>
<td>PCA 33 x 33</td>
<td>106,354</td>
<td>59</td>
<td>361</td>
<td>16,542</td>
</tr>
</tbody>
</table>

A stereographic approach was then used to compare the results. Fig. 6 presents the contour plots of all interramp slope orientations calculated on the DEM. The contour plots are based on a pole plot of the 106,354 poles representing slope orientation at each of the 106,354 cells of the DEM. Furthermore, the same contouring range was used for all figures, thus allowing easy comparison between stereonets. Intuitively, we should observe three preferential orientations corresponding to the east slopes, west slopes, and pit bottom since the north and south ends of the pit cover little area. The Unwin (1981), Horn (1982), and Sharpnack and Akin (1969) algorithms (Fig. 6a–c) do not provide pole densities corresponding to the designed interramp slopes. On the other hand, PCA analyses with sampling windows ranging from 5 x 5 to 33 x 33 cells reproduce the expected designed interramp angles. They also capture the inherent variability within slope orientations. Nevertheless, the clusters of orientation data are not identical for all PCA windows; in fact, they move slowly toward the center of the stereonet as the sampling window size increases. Until the sampling size reaches 13 x 13 cells the maximum pole concentration for the east and west walls is above the limiting value of 50° and 55°. This is not a plausible result since 50° and 55° were set as upper bound limits during the optimization process. For the largest sampling size (29 x 29 and 33 x 33 cells), the maximum pole concentration for both walls is well below these limiting angles, arguably suggesting that the derived slope orientations are more representative of a global slope angle rather than interramp angle. In other words, the window size is large enough to systematically contain a certain amount of horizontal cells defining a ramp, thus flattening the slope angle.
Fig. 6. Contour plot: (a) Unwin (1981), (b) Sharpnack and Akin (1969), (c) Horn (1982), (d) PCA 3 x 3, (e) PCA 5 x 5, (f) PCA 13 x 13, (g) PCA 21 x 21, (h) PCA 25 x 25, (i) PCA 29 x 29, (j) PCA 33 x 33.

Fig. 7 shows the slope dip direction histograms for all methods. Again, it can be seen that the Unwin (1981), Horn (1982), and Sharpnack and Akin (1969) algorithms cannot reproduce the trends observed within the pit. Furthermore, it seems that these methods, as well as the smaller sampling windows for PCA, are characterized by spikes in the distributions of dip direction values. Sampling windows at 13 x 13 cells and larger suggest well-defined bimodal distributions. These latter results are consistent with the optimized pit geometry.
Fig. 7. Dip direction histograms: (a) Unwin (1981), (b) Sharpnack and Akin (1969), (c) Horn (1982), (d) PCA 3 x 3, (e) PCA 5 x 5, (f) PCA 13 x 13, (g) PCA 21 x 21, (h) PCA 25 x 25, (i) PCA 29 x 29, (j) PCA 33 x 33.

Fig. 8 presents histograms for slope dips. As stated previously, the upper bound limit for the interramp dip is defined during the pit optimization process. This limit will dictate the upper bound value of interramp dip. Nevertheless, final dip values may be smaller depending on the orebody geometry. Furthermore, during final mine design stages, such as ramp design, the interramp angle can be altered, leading locally to steeper interramp slopes. From Fig. 8a–d one can tell that small sampling windows do not adequately capture expected concentrations in the slope dip values. These distributions are pseudo-uniform. At a window size approaching 13 x 13 cells, the dip histogram is uniform for shallow dip angles with a clear rise in the occurrence of dip values as we approach the
upper bound limits. For the 13 x 13 and 17 x 17 cells window, it can arguably be stated that a nonnegligible number of cells are defined by a dip angle higher than the limiting value of 50° or 55°. For the largest windows, 29 x 29 and 33 x 33, there is a transition towards shallower dip angles that could arguably be attributed to the intersection of some sampling windows with ramp sections. Furthermore, looking at the PCA results, one can note that there are very few readings with a dip between 0° and 5° corresponding to the pit floor. This is arguably attributable to the very narrow, elongated pit geometry.

Fig. 8. Dip histograms: (a) Unwin (1981), (b) Sharpnack and Akin (1969), (c) Horn (1982), (d) PCA 3 x 3, (e) PCA 5 x 5, (f) PCA 13 x 13, (g) PCA 21 x 21, (h) PCA 25 x 25, (i) PCA 29 x 29, (j) PCA 33 x 33.
A more formal way of comparing distributions is a Kolmogorov–Smirnov test (KS test). A 5% significance level was selected to accept or reject the null hypothesis that two distributions are from the same continuous distribution. This test was first used to compare dip direction distributions. All distributions obtained with the various algorithms were compared with the others. A total of 171 KS tests were performed. The results are presented in the lower portion of Table 2.

Table 2. KS test results for comparing dip (upper portion) and dip direction (lower portion) distributions (R indicates that the test was rejected and A that it was accepted).

|   | U | S | H | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 | 29 | 31 | 33 |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |

As expected, the smaller sampling window algorithms give dip direction distributions that do not correlate with each other or with the larger sampling window techniques, indicated by the R for a rejected correlation. As we reach a window of 19 x 19 cells, we can observe that the dip direction distributions are identical to the dip direction distributions of the neighboring window sizes, namely 17 x 17 and 21 x 21, and the test hypothesis was accepted (A). This trend is valid for all large windows greater than 17 cells or 42.5 m. Furthermore, for window sizes larger than 25 x 25 the dip direction distribution is identical to its four neighboring window sizes. This suggests that above a certain window size, the dip direction values reach a plateau and are less sensitive to a change in the sampling window.

The results of the dip distribution comparison using the KS test are presented in the upper part of Table 2. The test results indicate that dip distributions are constantly varying with the sampling size, and no plateau is reached. This can possibly be explained by the fact that for smaller window sizes the cell dimension is small compared to the size of the bench berms; thus a small variation in window size will greatly affect the dip angle by intersecting more cells along the berm. It can also be explained by the fact that for larger window sizes an increase in size will coincide with an increase in ramp section intersections, consequently also affecting the slope dip.

Another systematic method for comparing results commonly used in GIS approaches is the root mean square error (RMSE). This method is used on a cell-by-cell basis over the entire DEM. A resulting grid is compared to a reference grid to produce an output of differences between true and calculated slope orientation. RMS can be calculated for these differences in order to estimate the error in interramp orientation values. Usually, RMSE is computed for slope dip only. In this paper, to have a better representation of overall slope orientation, the differences are computed by measuring the acute angle between the pole of the true slope orientation and the pole of the calculated slope orientation. The acute angle is computed on a cell by cell basis according to

$$\cos \delta = l \cdot m$$  \hspace{1cm} (12)
where \( l \) is the vector defining the pole of the true orientation; \( m \) is the vector defining the pole of the evaluated orientation; and \( \delta \) is the acute angle between two poles.

For this paper, the true value for slope orientation is not known. From the previous sections it could arguably be stated that within a range of 17 x 17- and 25 x 25-cell windows, plausible interramp orientations are obtained. Accordingly, RMSE analyses were performed using three different grids for “true” orientation: 17 x 17, 21 x 21, and 25 x 25. The results are presented in Table 3. They indicate that small sampling windows show slope orientations diverging considerably from the true orientation values. The differences are inversely proportional to the window sampling sizes. RMSE analysis based on a 21 x 21 and 25 x 25 referential for true orientation indicate a difference of less than \( 3^\circ \) from their neighboring sampling sizes. This implies that an increase of 10 m in both \( x \) and \( y \) directions for the sampling window results in minimal and acceptable differences in slope orientation from a geomechanical perspective.

Table 3. RMS values for the acute angle \( \delta \) (interramp orientation).

<table>
<thead>
<tr>
<th>Referential grid</th>
<th>RMS ( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17 x 17</td>
</tr>
<tr>
<td>Unwin (1981)</td>
<td>28.82</td>
</tr>
<tr>
<td>Sharpnack and Akin (1969)</td>
<td>26.32</td>
</tr>
<tr>
<td>Horn (1982)</td>
<td>26.64</td>
</tr>
<tr>
<td>PCA 3 x 3</td>
<td>27.01</td>
</tr>
<tr>
<td>PCA 5 x 5</td>
<td>19.76</td>
</tr>
<tr>
<td>PCA 7 x 7</td>
<td>13.65</td>
</tr>
<tr>
<td>PCA 9 x 9</td>
<td>9.56</td>
</tr>
<tr>
<td>PCA 13 x 13</td>
<td>4.10</td>
</tr>
<tr>
<td>PCA 17 x 17</td>
<td>0</td>
</tr>
<tr>
<td>PCA 21 x 21</td>
<td>2.99</td>
</tr>
<tr>
<td>PCA 25 x 25</td>
<td>4.98</td>
</tr>
<tr>
<td>PCA 29 x 29</td>
<td>6.54</td>
</tr>
<tr>
<td>PCA 33 x 33</td>
<td>7.89</td>
</tr>
</tbody>
</table>

Based on the RMSE, KS test, histogram, and stereonet analysis results, it can arguably be stated that interramp slope orientations can be calculated adequately on the DEM of a final pit layout from PCA planar regression algorithms. For the Meadowbank case study, a window size of 21 x 21 cells, or 52.5 m, enables systematic and consistent representation of interramp orientation. Furthermore, the DEM layer for slope orientation offers a unique representation of the variability in orientation that can be observed on the pit topography. This type of information was not previously available for mine design purposes, and it could be the basis of more accurate and better integrated slope stability analysis.

A slope orientation map can be produced on a cell-by-cell basis, and this was elegantly done by Jaboyedoff et al. (2009). Slope orientation is defined by its pole and is color-coded according to a hue saturation intensity scheme applied on a stereonet (Fig. 9). Every possible slope orientation is thus expressed by a unique color. The color-coding methodology proposed by Jaboyedoff et al. (2009) was implemented in this paper by the authors and used to represent the geometry of the Portage pit. Fig. 10 gives the interramp orientation map for the Portage pit.
3.2.1 Bench level

An approach similar to the one used for interramp slopes was used to evaluate slope orientation at the bench level. The algorithms presented in Section 2 were used to assess slope orientation at that level. At bench level, dip angles are imposed by equipment and/or geomechanical and/or blasting considerations. Known a priori, these dip angles thus take a predetermined, fixed value. Accordingly, dip angles were not estimated, but were attributed a fixed value on a cell-by-cell basis on the DEM. These values were attributed based on the bench design angles presented in Fig. 5. Algorithms presented in Section 2 were thus used to assess only the dip direction of slope on the cells located on the crest of mining benches. Considering that cell size is smaller than bench height, this approach is providing a means of assessing locally, at a small scale, the dip direction of mining benches.

Fig. 11 shows the dip direction histograms studied at the bench face level. Fig. 11(a–e) is reproduced from Fig. 7(a–e) to enable direct visual comparison of all sampling window sizes analyzed at bench level. The 3 x 3-cell windows produce distributions characterized by spikes at specific dip direction values. This does not match the field data where no such preferential slope dip directions are planned. Starting from a window size greater than 5 x 5 cells, a smoother and more continuous bimodal distribution is observed, which is consistent with field data.
Fig. 11. Dip direction histograms: (a) Unwin, (b) Sharpnack and Akin, (c) Horn, (d) PCA 3 x 3, (e) PCA 5 x 5, (f) PCA 7 x 7, (g) PCA 9 x 9, (h) PCA 11 x 11, (i) 13 x 13.

The results of an RMSE analysis are presented in Table 4. The acute angle between the true pole value and the calculated pole value was again used for this analysis. RMSE analyses were performed using six different grids for true orientation: 3 x 3, 5 x 5, 7 x 7, 9 x 9, 11 x 11, and 13 x 13. At a referential window size of 3 x 3 cells it can be seen that there is a good agreement between a PCA 3 x 3 and the Sharpnack and Akin (1969) algorithm. This is a coherent result since they are, respectively, 9- and 8-cell planar regressions. Nevertheless, the results for the other KS tests are not indicative of an acceptable range at this sampling size. At window sizes greater than 9 x 9 cells, it can be observed that the differences between two consecutive window sizes are roughly within 4°.
This shows that for a window size greater than 9 x 9 cells, the bench dip direction is less sensitive to a change in sampling window sizes, and the results are more reproducible. Knowing that bench height is 20 m for the pit studied, it is arguably reasonable to use a sampling window of a similar size. A 9 x 9-cell or 22.5-m window was thus selected because it allows for consistent measurements of dip direction and it is in the bench height range. This type of variation in dip direction estimation is acceptable from a geomechanical perspective.

Table 4. RMS values for the acute angle $\delta$ (bench face orientation).

<table>
<thead>
<tr>
<th>Referential grid</th>
<th>RMS $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PCA 3 x 3</td>
</tr>
<tr>
<td>Sharpnack and Akin (1969)</td>
<td>0.23</td>
</tr>
<tr>
<td>PCA 3 x 3</td>
<td>0</td>
</tr>
<tr>
<td>PCA 5 x 5</td>
<td>10.39</td>
</tr>
<tr>
<td>PCA 7 x 7</td>
<td>14.55</td>
</tr>
<tr>
<td>PCA 9 x 9</td>
<td>16.87</td>
</tr>
<tr>
<td>PCA 11 x 11</td>
<td>18.27</td>
</tr>
<tr>
<td>PCA 13 x 13</td>
<td>19.22</td>
</tr>
</tbody>
</table>

Fig. 12 gives the map of bench face orientations. The same color coding presented in Fig. 9 was used to represent pole orientations. White indicates the horizontal surfaces typical of bench berms and pit bottoms. The scatter pole plot in the lower corner of the figure shows bench-face orientation for the entire pit, adequately representing the variability within the designed pit. One can see that while the majority of the bench faces dipping at $80^\circ$ are located on the west side of the pit, the variability within pit topography leads locally to some less-expected dip directions. The same conclusions may be drawn at the east wall, where benches dip at $70^\circ$. These local variations within slope orientations were not considered in previous stability analyses. The proposed approach provides a means of representing and considering all slope angles defining the final pit and also of locating them on the pit topography.
CONCLUSIONS

Standard practice in geomechanical rock slope engineering for open-pit mines relies on the simplified representation of slope geometry, which does not take full advantage of the available topographical data. This may lead to nonoptimal slope design. Furthermore, the geomechanical group often works “offline” with their own set of tools during the slope-creation process.

The primary objective of the paper was to present a methodology allowing for rigorous evaluations of interramp and bench slope orientations on the DEM of a designed open pit. The proposed approach leads to improved workflow in integrated slope creation through the use of a common software tool for all slope design aspects. First, the slope orientations obtained provide a rigorous assessment of slope orientation at both the interramp and the bench levels. Second, less frequent slope orientations within the final pit can be identified easily. Third, the various slope orientations can be located on a map of the pit, thus allowing an adequate spatial representation. Finally, subsequent slope stability analysis relying on those defined slope orientations offers a much better concordance between the as designed and the as-built slopes.

The methodology was validated with a case study: the Meadowbank Portage pit. The results suggest that for this particular pit, planar regression using PCA is better suited for slope orientation determination at both the interramp and the bench levels. The optimal sampling window for interramp was 21 x 21 cells while 9 x 9 cells are best suited at the bench level.

The methodology is flexible and depending on a given project’s block sizes and pit geometry, the optimal window size can be assessed, and slope orientation determined rapidly and consistently. Furthermore, the proposed methodology can consider a change in the initial planned pit topography both rapidly and efficiently. The DEM can be modified at any stage, based, for example, on new topographical information gathered through surveying, or modified based on new pit layouts dictated by new economic or production constraints. The new DEM will arguably be different from the previous one. Thus, slope orientations at both interramp and bench levels will be affected. A new slope orientation assessment will be performed. The initially defined window size will in most cases be adequate for the new pit geometry. Nevertheless, if the mining method is modified substantially,
for example, by using double or triple benching, the optimal sampling window can be reassessed rapidly using the proposed PCA planar regression analysis methodology.

ACKNOWLEDGEMENTS

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REFERENCES


