Unskilled traders, overconfidence and information acquisition

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Abstract

The value of an asset has two components, referred to as obvious and obscure. Unskilled traders are only aware of the obvious component and thus overestimate the precision of the information they may acquire. Unskilled traders are overconfident when informed and the intensity at which they trade makes researching the obscure component profitable to skilled traders, and this even when research costs are such that no information acquisition takes place in a skilled-only market. Hence overconfidence in our model encourages research by all, including the skilled (rational) traders.

Keywords: Overconfidence; Information acquisition; Market efficiency

JEL: D82; G12; G14

1. Introduction

Overconfidence is encountered in many spheres of human activity ([1, 2]). In trading models, it is generally represented as an underestimation of variance ([3]). When information acquisition is considered, overconfident traders are more likely to purchase information than rational traders, and the former may even crowd out the latter ([4, 5]). In this paper, we consider a trading model where some traders are overconfident because they are unskilled and unaware of it ([6, 7]). Unskilled traders have a limited knowledge of the factors affecting the asset and trade aggressively when informed. Rather than forcing skilled traders out of the market, the presence of unskilled traders may encourage them to gather information.
Our model builds on [8] with an asset that has two components, referred to as obvious and obscure. Information about each component can be acquired at a cost. Skilled traders know there are two components while the unskilled believe value to be entirely driven by the obvious component. An informed and unskilled trader overestimates the precision of his private information and trades more aggressively than would a skilled trader, thus revealing much information about the obvious component though the asset price. This encourages skilled traders to research the obscure component as they can then become almost fully informed by paying the obscure cost only. Hence contrary to [4] and [5], the presence of overconfident traders in our model generates rational (skilled) research.

The two types of traders in our model could be viewed as a representation of the dual-process hypothesis ([9, 10]), which suggests that there be fast and intuitive thinkers and slow and deliberative thinkers. [11] show that the former display overconfidence while the latter tend to be well calibrated. This setup differs from that of [12], who present a model where the unskilled is overconfident when he wrongly believes to be better than average, and where overconfidence fades away as one eventually learns his true type.

2. The model

An asset with a liquidation value \( \theta = A + B \) is traded by a continuum of agents located on the interval \([0, 1]\). \( A = a + \epsilon_a, B = b + \epsilon_b \) and the variables \( a, b, \epsilon_a \) and \( \epsilon_b \) are all normally and independently distributed: \( a \sim N(\bar{a}, \sigma_a^2), b \sim N(\bar{b}, \sigma_b^2), \epsilon_a \sim N(0, \sigma_{\epsilon_a}^2) \) and \( \epsilon_b \sim N(0, \sigma_{\epsilon_b}^2) \). A trader can, ahead of trading, learn the value of \( a \) at a cost \( c_a \) and the value of \( b \) at a cost \( c_b \).

A fraction \( m \) of market participants, referred to as unskilled, ignore the existence of \( B \) and believe that \( \theta = a + \epsilon_a \). Unskilled traders are identified by \( u \) and skilled traders, who are aware of both \( A \) and \( B \), are identified by \( s \). Unskilled and skilled traders simultaneously submit demand schedules, along with noise traders who submit \( z \sim N(0, \sigma_z^2) \), and a market-clearing price ensues. Each
trader has a CARA utility function given by $U(w) = -e^{-\rho w}$, with $\rho > 0$ and $w = (\theta - p)x - \tau_a c_a - \tau_b c_b$, where $x$ represents the position taken by the trader, $p$ is the clearing price, $\tau_a = 1$ if the trader learns $a$ and zero otherwise, and $\tau_b = 1$ if the trader learns $b$ and zero otherwise. Traders have zero initial positions and no budget constraints. Let $\mu_{a,j}$, $\mu_{b,j}$, $\mu_{ab,j}$ and $\mu_{n,j}$ denote the quantity of traders of type $j = s, u$ informed of $a$, $b$, both $a$ and $b$, and uninformed, respectively, with $\mu_{a,s} + \mu_{b,s} + \mu_{ab,s} + \mu_{n,s} = 1 - m$ and $\mu_{a,u} + \mu_{n,u} = m$.

**Definition 1.** An equilibrium is a set of trading functions $X_{i,j}(I, p)$, where $I_i \in \{\emptyset, a, b, \{a, b\}\}$, and a price $p$ such that, for all $i \in [0, 1]$ and $j = s, u$:

1. For each agent $i$ with information set $I_i$, $X_{i,j}(I_i, p)$ is a solution to

$$\max_x E_j \left[ -e^{-\rho((\theta - p)x)} \bigg| I_i, p \right] ;$$

2. The price $p$ is such that

$$\int_0^{1-m} X_{i,s}(I_i, p) di + \int_{1-m}^1 X_{i,u}(I_i, p) di + z = 0;$$

3. $\mu_{a,j}$, $\mu_{b,j}$, $\mu_{ab,j}$ and $\mu_{n,j}$ are such that

$$E_j \left[ -e^{-\rho((\theta - p)x_i,j(I_i, p) - c(I_i))} \right] \geq E_j \left[ -e^{-\rho((\theta - p)x_i,j(I_i', p) - c(I_i'))} \right]$$

for all $i \in [0, 1]$ and $j = s, u$, where $I_i, I_i' \in \{\emptyset, a, b, \{a, b\}\}$, $I_i' \neq I_i$

$c(a) = c_a$, $c(b) = c_b$ and $c(a, b) = c_a + c_b$.

As developed in Vives (2008), a trader of type $j = s, u$ prefers the information set $I$ over $I'$ if

$$e^{\rho(c(a) - c(b))} \frac{\text{Var}_j(\theta | I, p)}{\text{Var}_j(\theta | I', p)} < 1,$$  \hspace{2cm} (1)

where $\text{Var}_s$ denotes the correct variance and $\text{Var}_u$ denotes the variance from the viewpoint of an unskilled trader. Throughout the paper, we make the following assumption in order to reduce then number of cases to consider in equilibrium:

$$e^{\rho(c_a - c_b)} \frac{\text{Var}(\theta | a, p)}{\text{Var}(\theta | b, p)} = e^{\rho(c_a - c_b)} \frac{\sigma_a^2 + \sigma_{\epsilon_a}^2 + \sigma_{\epsilon_b}^2}{\sigma_a^2 + \sigma_{\epsilon_a}^2 + \sigma_{\epsilon_b}^2} < 1. \hspace{2cm} (2)$$
Condition 2 implies that a skilled trader finds learning $a$ more profitable than learning $b$ when all other traders are uninformed. This condition means that $a$ is relatively cheaper than $b$ in terms of uncertainty resolution. For example, $a$ could represent financial information easily accessible from the internet while $b$ could represent the talent and motivation of top employees and management, new projects, and so on. Factor $a$ is more obvious than $b$ as a factor influencing $\theta$, and is also cheaper to analyze.

We have ordered learning $a$ as being more cost-effective than learning $b$. We now order learning both $a$ and $b$ ahead of learning $a$ only:

$$e^{\rho (c_a + c_b - c_a)} \sqrt{\frac{\text{Var}(\theta|a,b,p)}{\text{Var}(\theta|a,p)}} = e^{\rho c_b} \sqrt{\frac{\sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}{\sigma^2_b + \sigma^2_a + \sigma^2_{\epsilon_b}}} < 1. \quad (3)$$

That is, when no one else is informed, a skilled trader prefers learning both $a$ and $b$ to learning $a$ alone. This condition rules out the case where a trader has to choose between learning $a$ or $b$. If $a$ is affordable from the viewpoint of a skilled trader, then becoming fully informed is also affordable.

**Lemma 1.** If inequalities 2 and 3 hold, then

$$e^{\rho c_a} \sqrt{\frac{\text{Var}(\theta|a,p)}{\text{Var}(\theta|p)}} = e^{\rho c_a} \sqrt{\frac{\sigma^2_{\epsilon_a}}{\sigma^2_a + \sigma^2_{\epsilon_a}}} < 1,$$

i.e. an unskilled trader always finds profitable to learn $a$ when all other traders are uninformed.

**Proof:** Inequality 2 yields

$$e^{\rho c_a} \sqrt{\frac{\sigma^2_{\epsilon_a}}{\sigma^2_a + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}} < e^{\rho c_b} \sqrt{\frac{\sigma^2_{\epsilon_b}}{\sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}} < e^{\rho c_b} \sqrt{\frac{\sigma^2_{\epsilon_b}}{\sigma^2_{\epsilon_b} + \sigma^2_{\epsilon_a}}} < 1,$$

where the last inequality stems from 3.

Note that Lemma 1 holds for any value of $\rho$, $\sigma^2_a$ and $\sigma^2_{\epsilon_a}$. It even holds when

$$e^{\rho (c_a + c_b)} \sqrt{\frac{\text{Var}(\theta|a,b,p)}{\text{Var}(\theta|p)}} = e^{\rho (c_a + c_b)} \sqrt{\frac{\sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}{\sigma^2_a + \sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}} > 1. \quad (4)$$

If 2, 3 and 4 hold, information is prohibitively costly to skilled traders and no information acquisition takes place in equilibrium in a skilled-only market. Introducing unskilled traders in such a market is then beneficial since it
generates research in component $a$. Research in component $b$ depends on the parameter values, as we will now see.

If conditions (2), (3) and (4) hold, then a fraction of unskilled traders become informed of $a$ and their demand function is given by (details in appendix)
\[
X_u(a, p) = \frac{1}{\rho \sigma^2_{\epsilon_a}} (a - p).
\]

Let $\alpha_{a,u} = \frac{1}{\rho \sigma^2_{\epsilon_a}}$ and let $\tilde{\alpha}_a = \mu_{a,u} \alpha_{a,u}$. Knowing that some unskilled traders learn $a$, a skilled trader is even less tempted to learn $a$ since then
\[
e^{\rho c_a} \sqrt{\frac{\text{Var}(\theta|a, p)}{\text{Var}(\theta|p)}} = e^{\rho c_a} \sqrt{\frac{\sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}{\sigma^2_a + \sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}} > e^{\rho c_a} \sqrt{\frac{\sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}{\sigma^2_a + \sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}}
\]
and, by (3) and (4), the last term of this inequality is greater than one. The same reasoning applies to learning both $a$ and $b$ versus remaining uninformed.

If, however, $\sigma^2_{\epsilon_a}$ is sufficiently small, then learning $b$ becomes interesting for a skilled trader. If $\mu_{a,u} > 0$ and no one knows $b$, a skilled trader considers
\[
e^{\rho c_b} \sqrt{\frac{\text{Var}(\theta|b, p)}{\text{Var}(\theta|p)}} = e^{\rho c_b} \sqrt{\frac{\sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}{\sigma^2_a + \sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}} < e^{\rho c_b} \sqrt{\frac{\sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}{\sigma^2_a + \sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}}
\]
As $\sigma^2_{\epsilon_a}$ approaches zero, $\tilde{\alpha}_a$ becomes arbitrarily large and the right-hand side of the last equation approaches $e^{\rho c_b} \sqrt{\frac{\sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}{\sigma^2_a + \sigma^2_b + \sigma^2_{\epsilon_a} + \sigma^2_{\epsilon_b}}} < 1$ given inequality (3). The proof of the following proposition can be found in appendix.

**Proposition 1.** Suppose (2), (3) and (4) hold. In the absence of unskilled traders ($m = 0$), all traders remain uninformed in equilibrium. If there is a positive fraction of unskilled traders ($m > 0$), then information acquisition in equilibrium is as follows:

1. None of the skilled traders learn $a$ alone nor both $a$ and $b$, i.e. $\mu_{a,s} = \mu_{ab,s} = 0$;
2. A positive number of unskilled traders learn $a$, with, in an interior solution,
\[
\mu_{a,u} = \rho \sigma^2 \sigma_{\epsilon_a} \sqrt{\frac{1}{e^{2\rho c_a} - 1} - \frac{\sigma^2_{\epsilon_a}}{\sigma^2_a}}.
\]
3. If $\sigma^2_a$ is small enough, a positive number of skilled traders learn $b$, where, in an interior solution $\mu_{b,s}$ is such that

$$e^{\rho cb} \sqrt{\frac{A + \sigma^2_a + \sigma^2_b}{\text{Var}(<\theta|p)>}} = 1$$

with

$$\text{Var}(<\theta|p>) = \frac{\sigma^2_a \sigma^2_b}{\tilde{\alpha}_a^2 \sigma^2_a + \tilde{\beta}_b^2 \sigma^2_b + \sigma^2_z} + \sigma^2_a + \sigma^2_b,$$

$$\tilde{\beta}_b = \frac{\mu_{b,s}}{\rho (A + \sigma^2_a + \sigma^2_b) + A \tilde{\alpha}_a \mu_{b,s} / \sigma^2_z},$$

$$\tilde{\alpha}_a = \frac{\mu_{a,u}}{\rho \sigma^2_{e_a}}, \text{ and } A = \frac{\sigma^2_a \sigma^2_b}{\tilde{\alpha}_a^2 \sigma^2_a + \sigma^2_z}.$$

Hence in a market where no information acquisition takes place when all traders are skilled, the introduction of unskilled traders generates research by both types of traders: Unskilled traders perform research in the component they are aware of, and skilled traders investigate the component overlooked by the unskilled. When informed, unskilled traders are overconfident as they overestimate the precision of their information. As in [4], the presence of overconfident traders may be needed in our model to generate research, but our model differs from theirs in that rational (skilled) traders may perform research because overconfident traders are present. In [5], the presence of overconfident traders reduces the research effort of rational traders in a way that may leave market efficiency untouched. Here the presence of overconfident traders does not reduce research by rational traders but increases it.

2.1. Example

This example shows how the presence of unskilled traders influences information acquisition decisions and improves market efficiency. Suppose that $\sigma_a = \sigma_b = \sigma_z = \rho = 1$, $\sigma_{e_b} = 0.1$, $c_a = 1$, $c_b = 2$ and $m = 0.2$. With these parameter values, inequalities (2), (3) and (4) hold and thus no traders become informed in the absence of unskilled traders and $\text{Var}(\theta|p) = \text{Var}(\theta)$. Table 1
displays the number of unskilled traders learning \( a \) and the number of skilled traders learning \( b \) with these parameter values, as well as the impact of information acquisition on the asset variance. When \( \sigma_{\epsilon a} \) approaches zero, few unskilled traders learn \( a \) but they trade so intensely that all skilled traders learn \( b \), thus significantly improving market efficiency. As \( \sigma_{\epsilon a} \) grows, more unskilled traders learn \( a \) but the intensity of their trades decreases as \( \sigma_{\epsilon a} \) increases, and this induces fewer skilled traders to learn \( b \). In this example, skilled traders do not learn anything when \( \sigma_{\epsilon a} \) exceeds one-third of \( \sigma_{\epsilon b} \), but information acquisition by unskilled traders keeps improving market efficiency.

| \( \sigma_{\epsilon a} \) | \( \mu_{a,u} \) | \( \mu_{b,a} \) | \( \text{Var}(\theta) \) | \( \text{Var}(\theta|p) \) | \( \sigma_{\epsilon a} \) | \( \mu_{a,u} \) | \( \mu_{b,a} \) | \( \text{Var}(\theta) \) | \( \text{Var}(\theta|p) \) |
|-----------------|---------------|---------------|-----------------|-----------------|-----------------|---------------|---------------|-----------------|-----------------|
| .001            | .0004         | .8000         | 2.0100          | .5464           | .025            | .0099         | .0269         | 2.0106          | .7981           |
| .002            | .0008         | .6236         | 2.0100          | .5476           | .030            | .0118         | .0112         | 2.0109          | .9091           |
| .003            | .0012         | .4146         | 2.0100          | .5496           | .031            | .0122         | .0082         | 2.0110          | .9337           |
| .004            | .0016         | .3097         | 2.0100          | .5524           | .032            | .0126         | .0053         | 2.0110          | .9591           |
| .005            | .0020         | .2465         | 2.0100          | .5561           | .033            | .0130         | .0024         | 2.0111          | .9853           |
| .010            | .0040         | .1177         | 2.0101          | .5863           | .034            | .0134         | .0000         | 2.0112          | 1.0074          |
| .015            | .0059         | .0717         | 2.0102          | .6368           | .100            | .0383         | .0000         | 2.0200          | 1.0639          |
| .020            | .0079         | .0456         | 2.0104          | .7074           | .200            | .0683         | .0000         | 2.0500          | 1.2556          |

Table 1: Number of unskilled traders knowing \( a \) and number of skilled traders knowing \( b \) when \( \sigma_a = \sigma_b = \sigma_z = \rho = 1, \sigma_{\epsilon a} = 0.1, c_a = 1, c_b = 2 \) and \( m = 0.2 \).

3. Conclusion

We develop a two-component version of \[8\] trading model with information acquisition, wherein we introduce unskilled traders who ignore the existence of one component. Even if parameter values are such that skilled traders do not find worthwhile to gather information when no one else is informed, unskilled traders may be interested in purchasing information about the one component they are aware of. When informed, unskilled traders are overconfident as they overestimate the precision of their information. The intensity with which un-
skilled and informed traders negotiate encourages skilled traders to perform research in the component overlooked by the unskilled. Thus introducing unskilled traders in a market where no one would otherwise do research may generate research in both asset components, thus improving market efficiency.

Appendix A. Proof of Proposition 1

Appendix A.1. Unskilled traders’ information acquisition decisions

Given a trader’s utility function, each order takes the form

\[ X_j(I, p) = \frac{E_j[\theta|I, p] - p}{\rho \text{Var}_j(\theta|I, p)} \] (A.1)

for \( I = a, b, \{a, b\}, \emptyset \) and \( j = s, u \). For an unskilled trader knowing \( a \), this gives \( X_u(a, p) = a - p \). Let \( \alpha_{a,u} = \frac{1}{\rho \sigma_a^2} \) and let \( \mu_{a,u} \) denote the number of unskilled traders who know \( a \). An unskilled trader believes all other traders to be unskilled as well and that the price will be given by

\[ \mu_{a,u} \alpha_{a,u} \left( a - \hat{p} \right) + \left( 1 - \mu_{a,u} \right) E_u[\theta|\hat{p}] - \hat{p} + \rho \text{Var}_u(\theta|\hat{p}) + z = 0. \]

The information an unskilled trader extracts from the price is \( \hat{\omega} = a + \frac{z}{\mu_{a,u} \alpha_{a,u}} \) and thus

\[ \text{Var}_{u}(\theta|p) = \text{Var}_{u}(\theta|\hat{\omega}) = \sigma_a^2 + \sigma_{\epsilon_a}^2 - \frac{\sigma_{\epsilon_a}^2}{\sigma_a^2 + \sigma_{\epsilon_a}^2} = \frac{\sigma_{\epsilon_a}^2 \sigma_a^2}{\sigma_a^2 (\mu_{a,u} \alpha_{a,u})^2 + \sigma_{\epsilon_a}^2 + \sigma_{\epsilon_a}^2}. \]

The number of unskilled traders learning \( a \) is then such that

\[ e^{\rho c_a \sigma_{\epsilon_a}} \sigma_{\epsilon_a} = \sqrt{\frac{\sigma_{\epsilon_a}^2 \sigma_a^2}{\sigma_a^2 (\mu_{a,u} \alpha_{a,u})^2 + \sigma_{\epsilon_a}^2 + \sigma_{\epsilon_a}^2}} \]

in an interior solution, which gives

\[ \mu_{a,u} = \rho \sigma_a \sigma_{\epsilon_a} \sqrt{\frac{1}{e^{2c_a} - 1} - \frac{\sigma_{\epsilon_a}^2}{\sigma_a^2}}. \]

Appendix A.2. Skilled trader’s information acquisition decision

Suppose \( \mu_{a,u} > 0 \) and let \( \tilde{\alpha}_a = \mu_{a,u} \alpha_{a,u} \). If

\[ e^{\rho c_b \sqrt{\frac{\sigma_{\epsilon_a}^2 \sigma_a^2}{\sigma_a^2 + \sigma_{\epsilon_a}^2} + \sigma_{\epsilon_b}^2} - \frac{\sigma_{\epsilon_a}^2}{\sigma_a^2 + \sigma_{\epsilon_a}^2} < 1}, \]
then some skilled traders learn the value of $b$. Let $\mu_{b,s}$ denote the number of skilled traders informed of $b$. We have shown earlier that $X_u(a, p) = \alpha_{a,u}(a - p)$.

Suppose that the order of a skilled trader informed of $b$ is given by $X_s(b, p) = \beta_{b,s}b - \beta_{p,s}p + \beta_s$, and that $X_u(p) = -\gamma_{p,s}p + \gamma_s$ and $X_u(p) = -\gamma_{p,u}p + \gamma_u$ represent the order submitted by an uninformed skilled and an uninformed unskilled, respectively. Let $\tilde{\beta}_b = \mu_{b,s}\beta_{b,s}$, $\tilde{\beta}_p = \mu_{b,s}\beta_{p,s}$, $\tilde{\gamma}_p = (m - \mu_{a,u})\gamma_{p,u} + (1 - m - \mu_{b,s})\gamma_{p,s}$ and let $\tilde{\gamma} = (m - \mu_{a,u})\gamma_u + (1 - m - \mu_{b,s})\gamma_s$. Then the market clearing price is given by

$$p = \lambda \left( \tilde{\alpha}_a a + \tilde{\beta}_b b + \tilde{\beta} + \tilde{\gamma} + z \right),$$

where $\lambda = (\tilde{\alpha}_a + \tilde{\beta}_p + \tilde{\gamma}_p)^{-1}$. The order of a skilled trader informed of $b$ then develops into

$$X_s(b, p) = \frac{\tilde{\alpha}_a a^2 + \sigma_a^2}{\rho(\sigma_a^2 + (\sigma_a^2 + \tilde{\alpha}_a a^2 + \sigma_a^2))} (b - p) + \frac{\sigma_b^2}{\rho(\sigma_a^2 + (\sigma_a^2 + \tilde{\alpha}_a a^2 + \sigma_a^2))} \pi$$

$$+ \frac{\tilde{\alpha}_a a^2}{\rho(\sigma_a^2 + (\sigma_a^2 + \tilde{\alpha}_a a^2 + \sigma_a^2))} \left( \frac{p}{\lambda} - (\tilde{\beta}_b + \tilde{\beta} + \tilde{\gamma}) \right),$$

and thus

$$\beta_{b,s} = \frac{\tilde{\alpha}_a a^2 + \sigma_a^2 - \tilde{\alpha}_a a^2 \mu_{b,s} \beta_{b,s}}{\rho(\sigma_a^2 + (\sigma_a^2 + \tilde{\alpha}_a a^2 + \sigma_a^2))}.$$

This gives

$$\beta_{b,s} = \frac{\tilde{\alpha}_a a^2 + \sigma_a^2}{\rho(\sigma_a^2 + (\sigma_a^2 + \tilde{\alpha}_a a^2 + \sigma_a^2))} + \tilde{\alpha}_a a^2 \mu_{b,s} = \frac{1}{\rho(A + \sigma_a^2 + \sigma_{\tilde{\alpha}_a}^2) + A\tilde{\alpha}_a \mu_{b,s}/\sigma_{\tilde{\alpha}_a}^2},$$

where $A = \frac{\sigma_a^2 \sigma_b^2}{\sigma_a^2 + \sigma_b^2}$. In an interior solution $\mu_{b,s}$ is such that

$$e^{\rho c_b} \sqrt{\frac{A + \sigma_a^2 + \sigma_{\tilde{\alpha}_a}^2}{\text{Var}(\theta|p)}} = 1,$$

where

$$\text{Var}(\theta|p) = \frac{\sigma_a^2 \sigma_b^2 (\tilde{\alpha}_a - \tilde{\beta}_b)^2 + \sigma_a^2 (\sigma_a^2 + \sigma_{\tilde{\alpha}_a}^2)}{\tilde{\alpha}_a^2 \sigma_a^2 + \beta_{b,s}^2 \sigma_b^2 + \sigma_a^2} + \sigma_b^2 + \sigma_{\tilde{\alpha}_a}^2$$

and

$$\tilde{\beta}_b = \frac{\mu_{b,s}}{\rho(A + \sigma_a^2 + \sigma_{\tilde{\alpha}_a}^2) + A\tilde{\alpha}_a \mu_{b,s}/\sigma_{\tilde{\alpha}_a}^2}.$$
References


